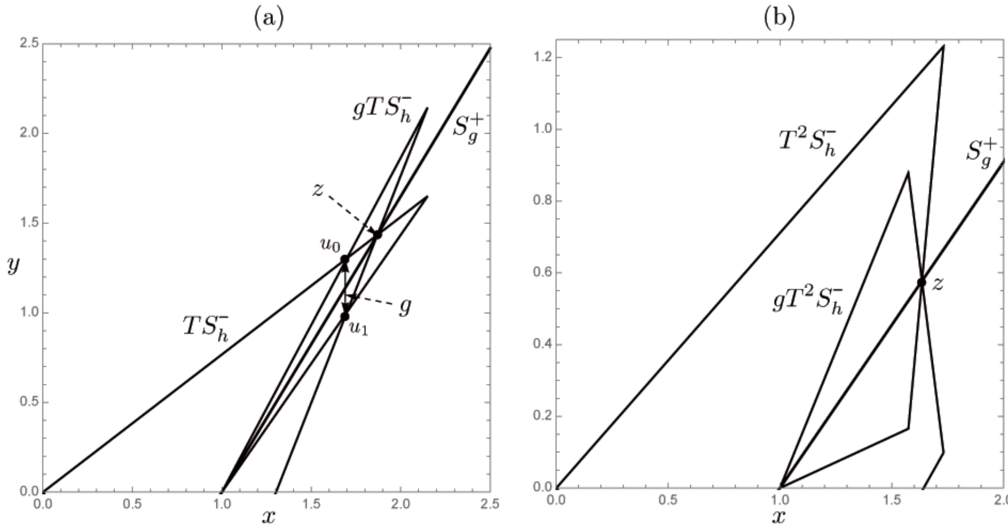

Fig. 2. (a) This figure represents the relation between the subdominant axis TS_h^- and its image gTS_h^- at $a = 3.3 > a_c(1/3) = 3$. The intersection point z is an orbital point of 1/3-BE. The relation $u_1 = gu_0$ holds. (b) This figure represents the relation between the subdominant axis $T^2S_h^-$ and its image $gT^2S_h^-$ at $a = (\sqrt{5}+1)/2 + 0.2 = 1.818033\ldots$. The intersection point z is an orbital point of 1/5-BE.

From Definition 1 and Theorem 2, the branches of symmetry axes on which p/q-BE(BS) has a point are determined. The results are summarized as Property 3 (Yamaguchi and Tanikawa, 2009).

Property 3.
(i) If q and p are odd, then p/q-BE has one orbital point on S_g^+ and another on S_h^-, while p/q-BS has one orbital point on S_g^- and another on S_h^+.
(ii) If q is odd and p is even, then p/q-BE has one orbital point on S_g^+ and another on S_h^+, while p/q-BS has one orbital point on S_g^- and another on S_h^-.
(iii) If q is even and p is odd, then p/q-BE has one orbital point on S_g^+ and another on S_g^-, while p/q-BS has one orbital point on S_h^+ and another on S_h^-.

From now, we discuss the properties of involutions. Suppose that curve $y = G(x)$ intersects S_g at $z = (x, y)$. Let $\xi(z) = dG(x)/dx$ be the slope of the curve at z. Operating g to this curve, we obtain the image curve.

$$y = -G(x) - f(x). \tag{6}$$

Let $\xi_g(z) = dy/dx$ be the slope of the image curve at z. We obtain the relation

$$\xi_g(z) = -\xi(z) - f'(x) \tag{7}$$

where $f'(x) = df(x)/dx$. There are two situations in which $\xi_g(z)$ and $\xi(z)$ coincide at $z \in S_g$. In the first case, both $\xi_g(z)$ and $\xi(z)$ diverge. In the second case, the relations $\xi(z) = \xi_g(z) = -f'(x)/2$ hold where $-f'(x)/2$ is the slope of S_g at z.

Next, suppose that the curve represented by $y = H(x)$ intersects S_h at $w = (x, 0)$. Let $\eta(w) = dH(x)/dx$ be the slope of the curve at w. Operating h to this curve, we have $hH(x)$.

$$y = -H(x - y). \tag{8}$$

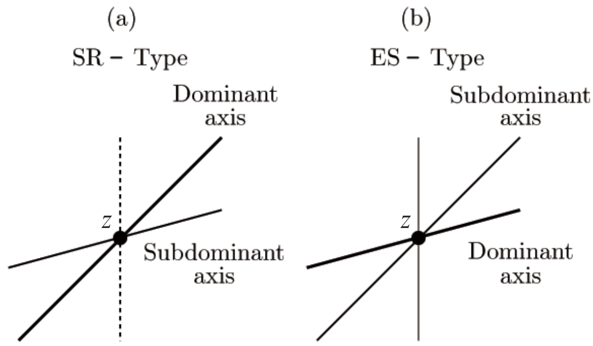

Fig. 3. Two types of the intersection of the dominant axis and the subdominant axis. (a) SR-Type (abbreviation of saddle with reflection) where $\xi(z) < \xi_{\mathrm{D}}(z)$ holds. (b) ES-Type (abbreviation of elliptic or saddle) where $\xi(z) > \xi_{\mathrm{D}}(z)$ holds. This type includes the situation that the slope of the subdominant axis diverges.

Let $\eta_h(w) = dy/dx$ be the slope of $hH(x)$ at w. We obtain the relation

$$\eta_h(w) = \frac{\eta(w)}{\eta(w) - 1}. \tag{9}$$

There exists the situation that the function $H(x)$ and its image $hH(x)$ are tangent at $w \in S_h$. At the tangency situation, the following relations hold.

$$\eta(w) = \eta_h(w) = 0 \ \text{ or } \ \eta(w) = \eta_h(w) = 2. \tag{10}$$

2.3 Involutions and symmetry axes for T^q

Mapping T^q is also reversible. In fact, it can be represented by a product of two involutions. Here let us define the subdominant axis which makes a pair with the dominant axis.

Definition 4 (Subdominant axis). Mapping T^q $(q \geq 1)$ is represented as follows.

$$T^q = T^{q-1}h \circ g. \tag{11}$$