Table 13. Universal hypocycloid used as axes and hypocycloid used as motifs ( $a_3 = 12$ ,  $b_3 = 2$ ,  $c_3 = 1$ ,  $a_2 = 3\tau$ ,  $b_2 = 1$ ,  $f_1 : 0 \sim 640\pi$ ,  $f_2 : 0 \sim 730\pi$ ).



Fig. 18. Geometric patterns using combinations of universal hypocycloid used as axes and hypocycloid used as motifs.

Figure 18c shows a Type A geometric pattern using hypocycloid as a motif. Figure 18d shows a Type B geometric pattern using hypocycloid as a motif.

## 4. Discussion

We generated geometric patterns using four types of geometric curves as the axes and three types of geometric curves as motifs. The key points of the method described in this paper are as follows: 1) We devised equations corresponding to the Type A and Type B geometric patterns devised to determine geometric shapes. 2) We devised the coefficient equation corresponding to Type A geometric patterns  $(s_1 = d_1 \pi / J)$ , and the coefficient equation corresponding to Type B geometric patterns ( $s_2 = d_2 \pi/J$ ). 3) We devised the equations  $h = m2^n j$  (*m*: natural number; *n*: integer) and  $d_2 = m2^n d_1$ , which are used when determining the values of the h, j,  $d_1$  and  $d_2$  coefficients that determine the pattern shape. We used these key points to devise a methodology for generating geometric patterns. We investigated the effectiveness of this methodology. Our findings can be summarized as follows:

## 4.1 Determining factors for pattern generation and pattern features

The shape of the generated pattern is determined by the values of the coefficients h, j,  $d_1$ , and  $d_2$ . We generated several patterns using a wide range of different values for each of the coefficients and determined that the values below are appropriate for our method. Type A and Type B geometric pattern shape differences are determined by the combinations of values used for the h and j, and  $d_1$  and  $d_2$  coefficients. To set the  $d_1$  and  $d_2$  coefficients, we set  $d_1$  for Type A geometric patterns and  $d_1$  for Type B geometric patterns to the same value or to different values.

## a. Using universal circle as axes

When using a universal circle as the axes: To set the h and j coefficients, we determined the value of h by setting

 $j = 40, 50 \text{ or } 60; m = 1 \text{ and } n = -1 \text{ for Type A geometric patterns. To set the } d_1 \text{ and } d_2 \text{ coefficients, we determined value of } d_2 \text{ by setting } d_1 = 0.5 \text{ or } 1; m = 1; \text{ and } n = -1 \text{ or } 1 \text{ for Type B geometric patterns.}$ 

With golden ellipse as motif: Figure 7c shows an example of a generated Type A geometric pattern for which  $d_1 = 1$ . The pattern in Fig. 7c is a convex curve in the shape of the pattern opening. Figure 7d shows an example of a generated Type B geometric pattern for which  $d_1 = 1$  and  $d_2 = 0.5$ . The pattern in Fig. 7d is a convex curve with three cusps in the shape of the pattern opening. Varying the value of *m* to 3, 4, and 5 at inflection point  $d_2$  enables generation of variations in the shape of the pattern opening.

With epicycloid as a motif: The curve shown in Fig. 8b is a closed curve with two inflection points. The pattern's internal shape is therefore determined by the number of inflection points, in contrast to a golden ellipse. Figure 8c shows an example of a Type A geometric pattern generated with  $d_1 = 1$ . Figure 8d shows an example of a Type B geometric pattern generated with  $d_1 = 1$  and  $d_2 = 0.5$ . It is considered that the shape of the pattern opening of Fig. 8c is a convex curve with two inflection points and the shape of the pattern opening Fig. 8d is a convex curve with three inflection points.

With hypocycloid as a motif: The curve shown in Fig. 9b is a closed curve with three inflection points. Figure 9c shows an example of a Type A geometric pattern generated with  $d_1 = 0.5$ . Figure 9d shows an example of a Type B geometric pattern generated with  $d_1 = 0.5$  and  $d_2 = 1$ . The visual outline of the pattern of Fig. 9d is a closed curve with nine inflection points.

## b. Using universal golden ellipse as axes

When using a universal golden ellipse as the axes: To set the *h* and *j* coefficients, we determined the value of *h* by setting j = 30, or 40; m = 1 and n = 0. To set the