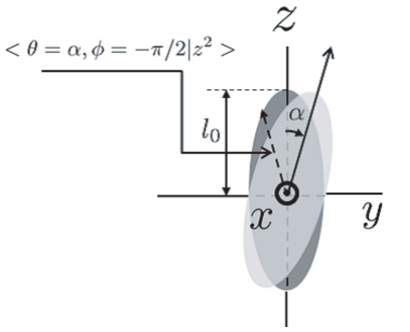

Fig. 6. Method of obtaining the form of $|z^2>$.

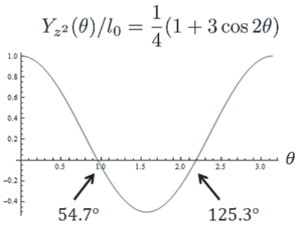

Fig. 7. Functional form of $< \theta|z^2 >$.

Let us examine the z direction. We multiply the "bra" $< \theta = 0|$ to both sides of Eq. (96).

$$
\begin{aligned}
&< \theta = 0|e^{i\mathbf{L_x}\alpha}|z^2 >= \sqrt{2}C(-\alpha) < \theta = 0|x^2 - y^2 > \\
&+\sqrt{2}G(-\alpha) < \theta = 0|yz > \\
&+J(-\alpha) < \theta = 0|z^2 > . \qquad (97)
\end{aligned}
$$

Note that $< \theta = 0|x^2-y^2 >=< \theta = 0|yz >= 0$ hold here. This comes from the following reasons. From Eq. (93) and Fig. 5, we have $< \theta = 0|x^2 - y^2 >= 0$. From Eq. (91) and Fig. 5, we have $< \theta = 0|yz >= 0$. Both are later shown in Fig. 9 explicitly. Furthermore, from

$$
e^{-i\mathbf{L_x}\alpha}|\theta = 0 >= |\theta = \alpha, \phi = -\pi/2 >,
$$

we obtain

$$
\begin{aligned}
< \theta = 0|e^{i\mathbf{L_x}\alpha}|z^2 > &= < \theta = \alpha, \phi = -\pi/2|z^2 > \\
&= < \theta = \alpha|z^2 >, \qquad (98)
\end{aligned}
$$

where the final equality originates from the rotational symmetry around the z axis from Eq. (94). Then we obtain

$$
< \theta = \alpha|z^2 >= J(-\alpha) < \theta = 0|z^2 > . \qquad (99)
$$

In an explicit form, we have

$$
Y_{z^2}(\theta, \phi) = \frac{l_0}{4}(1 + 3\cos 2\theta), \quad l_0 \equiv Y_{z^2}(\theta = 0). \qquad (100)
$$

This method is graphically shown in Fig. 6. The dark gray ellipsoid shows the $|z^2>$ state and the light gray ellipsoid shows the state $|z'^2> \equiv e^{i\mathbf{L_x}\alpha}|z^2>$. Then, we easily find that

$$
< \theta = \alpha, \phi = -\pi/2|z^2 >=< \theta = 0|z'^2 > .
$$

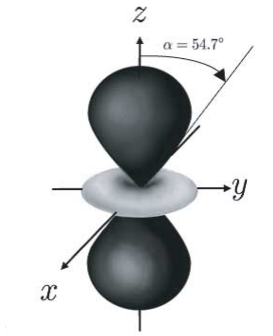

Fig. 8. Form of $< \theta|z^2 >$.

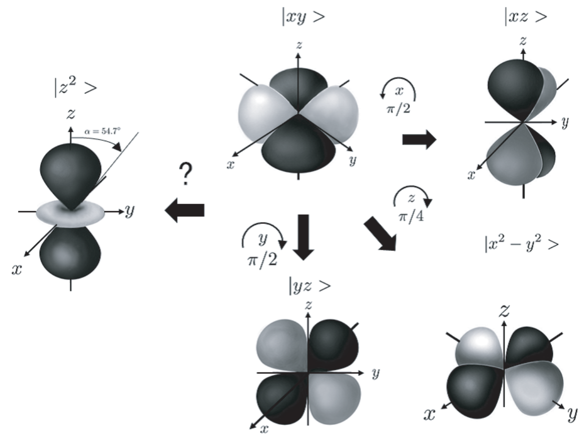

Fig. 9. Forms of the members of $l = 2$.

(The length of the dashed arrow of $|z^2>$ is the same as the length of $|z'^2>$ in the z direction.) Furthermore, $|z'^2>$ can be expanded in the form of Eq. (96). We therefore obtain Eq. (100). From this result, we have the form of $< \theta|z^2 >$ shown in Fig. 7.

In Fig. 8, the dark gray part and light gray part (similar to a torus but with a point hole) have opposite phases. The node plane becomes two cones with $\theta = 54.7°$ and $\theta = 125.3°$.

The states comprising the members of $l = 2$ are shown in Fig. 9. One of the remaining problems is the relation between $|z^2>$ and the other states. From Fig. 9, we search for the states that may become elements to construct the $|z^2>$ state. The rotation of $|xz>$ around the y axis by $-\pi/4$ with the rotation of $|yz>$ around the x axis by $\pi/4$ may have similar forms to $|z^2>$ as shown in Fig. 10.

This idea can be realized in the following calculation:

$$
\begin{aligned}
&e^{i\mathbf{L_y}\pi/4}|xz > +e^{-i\mathbf{L_x}\pi/4}|yz > \\
&= (-\frac{1}{2}|x^2 - y^2 > +\frac{\sqrt{3}}{2}|z^2 >) \\
&\quad +(\frac{1}{2}|x^2 - y^2 > +\frac{\sqrt{3}}{2}|z^2 >) \\
&= \sqrt{3}|z^2 > .
\end{aligned}
$$