

Fig. 1. Conduction band model  $\varepsilon_k$  for  $k_y = 0$ .



Fig. 2. Magnetic ordered structures of  $\langle S_n \rangle$ : (a) ferromagnet (Q = 0,  $\theta = \pi/2$ ); (b) helix ( $Q \neq 0$ ,  $\theta = \pi/2$ ); (c) cone ( $Q \neq 0$ ,  $0 < \theta < \pi/2$ ); (d) ferromagnet ( $Q \neq 0$ ,  $\theta = 0$ ).

with a cone angle  $\theta$  and a helical wave vector  $\boldsymbol{Q}$  along the  $k_z$  axis as shown in Fig. 2 [3],

$$\langle S_n \rangle = \langle S \rangle (\sin \theta \cos \phi_n, \sin \theta \sin \phi_n, \cos \theta).$$
 (5)

where  $\phi_n = (\mathbf{Q} \cdot \mathbf{R}_n)$  is the turn angle. This cone structure becomes the helical structure for  $\theta = \pi/2$  and the ferromagnetic structure for  $\theta = 0$  or  $\mathbf{Q} = \mathbf{0}$ .

If there is no crystalline anisotropy, the cone axis can be in any direction. For an anisotropy of easy plane the cone axis is the *c*-direction while for an anisotropy of easy *c*-axis, the situation is more complicated [5,20].

For the first transformation the local axes  $(\xi_n, \eta_n, \zeta_n)$  are introduced at each site  $\mathbf{R}_n$ , where  $\zeta_n$  is defined to be along the direction of the local moment  $\langle \mathbf{S}_n \rangle$ ,  $\xi_n$  along the direction perpendicular to both  $\zeta_n$  and z axes, and  $\eta_n$  along the direction perpendicular to  $\zeta_n$  and  $\eta_n$  axes as shown in Fig. 3. Those unit vectors  $\mathbf{e}_{n\xi}$ ,  $\mathbf{e}_{n\eta}$ ,  $\mathbf{e}_{n\zeta}$  are given by

$$e_{n\xi} = (\cos\theta \cos\phi_n, \cos\theta \sin\phi_n, -\sin\theta), e_{n\eta} = (-\sin\phi_n, \cos\phi_n, 0), e_{n\zeta} = (\sin\theta \cos\phi_n, \sin\theta \sin\phi_n, \cos\theta).$$
(6)



Fig. 3. Local coordinate axes  $(\xi_n, \eta_n, \zeta_n)$ .

Hence the *f*-spin operator  $S_{n\xi}$ ,  $S_{n\eta}$ ,  $S_{n\zeta}$  are defined by

$$\mathbf{S}_n = S_{n\xi} \ \mathbf{e}_{n\xi} + S_{n\eta} \ \mathbf{e}_{n\eta} + S_{n\zeta} \ \mathbf{e}_{n\zeta}. \tag{7}$$

The unperturbed Hamiltonian  $H_0$  is diagonalized by the transformation,

$$A_{\boldsymbol{k}-} = a_{\boldsymbol{k}\uparrow} \cos\theta_{\boldsymbol{k}} + a_{\boldsymbol{k}-\boldsymbol{Q}\downarrow} \sin\theta_{\boldsymbol{k}} A_{\boldsymbol{k}+} = -a_{\boldsymbol{k}\uparrow} \sin\theta_{\boldsymbol{k}} + a_{\boldsymbol{k}-\boldsymbol{Q}\downarrow} \cos\theta_{\boldsymbol{k}}$$
(8)

with

$$\cos(2\theta_{k}) = \frac{\varepsilon_{k} - \mathbf{Q} - \varepsilon_{k} + 2y}{\sqrt{(\varepsilon_{k} - \mathbf{Q} - \varepsilon_{k} + 2y)^{2} + 4x^{2}}}$$
(9)

where  $0 \le \theta_k \le \frac{\pi}{2}$  and

$$x = I\langle S \rangle \sin \theta, \quad y = I\langle S \rangle \cos \theta.$$
 (10)

After these two transformations, the Hamiltonian  $H_1$  is rewritten as

$$H = H_{0} + H_{1}$$

$$H_{0} = \sum_{\boldsymbol{k}\mu} E_{\boldsymbol{k}\mu} A^{\dagger}_{\boldsymbol{k}\mu} A_{\boldsymbol{k}\mu}$$

$$H_{1} = -IN^{-1} \sum_{\boldsymbol{k}} \sum_{\boldsymbol{k}'} \sum_{\boldsymbol{n}} e^{i(\boldsymbol{k}-\boldsymbol{k}') \cdot \boldsymbol{R}_{n}}$$

$$\times (A^{\dagger}_{\boldsymbol{k}-} A^{\dagger}_{\boldsymbol{k}+})(\sigma_{-}(\boldsymbol{k}, \boldsymbol{k}')S_{n+}$$

$$+\sigma_{+}(\boldsymbol{k}, \boldsymbol{k}')S_{n-} + \sigma_{\zeta}(\boldsymbol{k}, \boldsymbol{k}')S_{n0})$$

$$\times \begin{pmatrix} A_{\boldsymbol{k}'-} \\ A_{\boldsymbol{k}'+} \end{pmatrix}$$

$$(11)$$

where the new dispersion energy of the conduction electron  $E_{ku}$  is given by (see Fig. 10)

$$E_{k\pm} = \frac{1}{2} \left[ \varepsilon_{k} + \varepsilon_{k-Q} \\ \pm \sqrt{(\varepsilon_{k-Q} - \varepsilon_{k} + 2y)^{2} + 4x^{2}} \right].$$
(12)

The *f*-spin operators  $S_{n\pm}$  and  $S_{n0}$  are

$$S_{n\pm} = S_{n\xi} \pm iS_{n\eta}, \quad S_{n0} = S_{n\zeta} - \langle S \rangle, \quad (13)$$

and the new spin matrices  $\sigma_{\pm}(\mathbf{k}, \mathbf{k}')$  and  $\sigma_{\zeta}(\mathbf{k}, \mathbf{k}')$  operating on the pseudo spin  $\pm$  in  $A_{\mathbf{k}\pm}$  are

$$\sigma_{-}(\mathbf{k}, \mathbf{k}') = \frac{1}{2} [\sigma_{3} \sin(\theta_{\mathbf{k}} + \theta_{\mathbf{k}'} - \theta) + \sigma_{1} \cos(\theta_{\mathbf{k}} + \theta_{\mathbf{k}'} - \theta) + \sigma_{1} \cos(\theta_{\mathbf{k}} - \theta_{\mathbf{k}'}) + 1 \sin(\theta_{\mathbf{k}} - \theta_{\mathbf{k}'}) + 1 \sin(\theta_{\mathbf{k}} - \theta_{\mathbf{k}'})],$$

$$\sigma_{+}(\mathbf{k}, \mathbf{k}') = \sigma_{-}(\mathbf{k}', \mathbf{k})^{\dagger},$$

$$\sigma_{\zeta}(\mathbf{k}, \mathbf{k}') = \sigma_{3} \cos(\theta_{\mathbf{k}} + \theta_{\mathbf{k}'} - \theta) - \sigma_{1} \sin(\theta_{\mathbf{k}} + \theta_{\mathbf{k}'} - \theta),$$

$$(14)$$