Table 1. Spin values S, reduced crystal-field energy $\widetilde{V} = V_2^0 J^2/E_f$, reduced c-f exchange energy Δ_0 at 0 K, and reduced life-times Γ's: $\Gamma(T_N)$ at T_N, $\Gamma(T_C)$ at T_C, Γ_0 at 0 K. $E_f = 0.24$ eV, $I = 0.097$ eV and $\tilde{k}_a = 1.8$. See Eq. (20).

	S	$\widetilde{V}$	Δ_0	$\Gamma(T_N)$	$\Gamma(T_C)$	Γ_0
Gd	7/2	–	1.4	–	0.40	0.20
Tb	3	9.3	1.2	0.30	0.29	0.15
Dy	5/2	10.4	1.0	0.25	0.18	0.15
Ho	2	3.3	0.8	0.20	0.10	0.10
Er	3/2	-6.3	0.6			
Tm	1	-12.3	0.4			

As v_1 is estimated to be 3.2×10^{-7} cm/s and the ratio v_2/v_1 is to be about 0.25 from the band calculation of Dy [6], C is estimated to be about 300. Hence at $T = 0$, the spin-wave constant D_1 is estimated to be 0.0125 or 0.0147 eVÅ^{-2} and the maximum magnon-energy $\omega_{\max} = 2SJ_+(0)$ to be 14.1 or 15.7 meV for $\Gamma = 0$ or 0.2, respectively. The experimental values of D_1 and $\omega_{\max}$ are 0.0245 eVÅ^{-2} and 14.3 meV, respectively [10]. A discrepancy between those two values of D_1 should be due to neglect of the other part of Fermi surfaces in the present model. By assuming that the value of Δ is 1.4 or 1.0, and the value of Γ is 0.2 or 0.3 for $T/T_C = 0$ or 0.7, respectively, we can calculate magnon dispersions of ferromagnetic Gd and compare those with the experimental dispersions as shown in Fig. 15 [10]. From the above parameters the characteristic wavenumber $Q_c = 2E_f/v_1$ is estimated to be $0.21[2\pi/c]$, where c is the lattice constant along the c-axis. The anomaly of temperature dependence of the magnon dispersions in the region of $q_z < Q_c$ is explained by the softening of the magnon energies. The reason is that, as mentioned in Ref. [20], decreasing Δ or with increasing temperature, the ferromagnetic state passes near the ferro-cone or ferro-helix boundary (see Fig. 14). Hence we may expect that the softening of the magnon dispersion results in an anomalous decrease of the magnetization [31]. A discrepancy in large q_z region should be due to neglect of effects of other Fermi surfaces, the $\boldsymbol{k}$-dependence of the c-f exchange matrix element I and the zone boundary effect [32].

Secondly we consider the magnon dispersion of Ho in the helical phase. In the helix, the spin-vectors $\langle \boldsymbol{S}_n \rangle$ rotate as their positions $\boldsymbol{R}_n$ advance in the direction of $\boldsymbol{Q}_0$, and are parallel in a particular basal plane perpendicular to the c axis due to the axial crystal-field-anisotropy. Hence the magnon energy at $q_z = Q_0$ becomes finite. Figure 16 shows observed magnon dispersions at 50 K and 78 K [14]. The effect of the axial anisotropy should be introduced to the frequency $\omega(\boldsymbol{q})$ as

$$\omega(\boldsymbol{q}) = \sqrt{[F_1(\boldsymbol{q}) + 2SB]\, F_2(\boldsymbol{q})} \tag{73}$$

where S is the total angular momentum and B the effective axial-anisotropy constant. The constant B is given by (see Appendix D)

$$S^2 B = \frac{3}{2}\,\overline{V_2{}^0} - \frac{15}{4}\,\overline{V_4{}^0} + \frac{105}{16}\,\overline{V_6{}^0}, \tag{74}$$

in which $\overline{V_l{}^0}$ is the axial-anisotropy constant of the l-th order. From both experimental fit and theoretical fit by assuming the point-charge model, $\overline{V_2{}^0}$, $\overline{V_4{}^0}$ and $\overline{V_6{}^0}$ have been evaluated to be 2.66, 0.414 and -0.539 meV, respectively [1]. The temperature dependence of $\overline{V_n{}^0}(T)$ is given by the relation

$$\overline{V_n{}^0}(T) = \overline{V_n{}^0}\, \sigma^{n(n+1)/2},$$

and the reduced magnetization $\sigma = \langle S \rangle / S$ is derived from the neutron scattering in Ho [33]. By using Eq. (74) and the relation of $\overline{V_n{}^0}(T)$, the value of B is estimated to be 0.032 meV at 50 K and 0.023 meV at 78 K.

The model-parameters $\tilde{k}_a$ and Δ_0 are assumed to be 1.8 and 0.6, respectively. These values can reproduce the temperature dependence of helical Q_0 [20, 34], but this value of Δ_0 is a little smaller than the value in Table 1. By use of an equation $\Delta/\Delta_0 = \sigma$, Δ is 0.53 at 50 K and 0.4 at 78 K. From the fact that the wavenumber Q_0 approaches 0.27 $[2\pi/c]$ at $T_N = 133$ K, E_f/v_1 is estimated to be 0.14 $[2\pi/c]$. For simplicity we put $\Gamma = 0$.

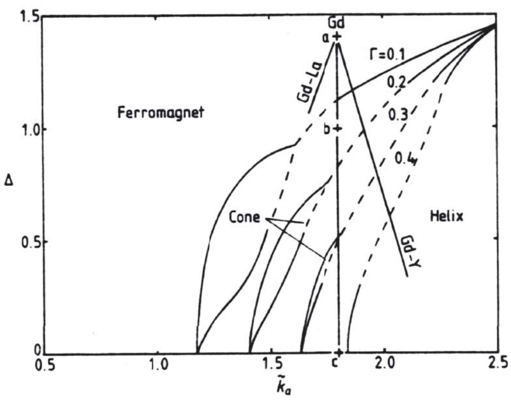

Fig. 14. Phase diagram for $V = 0$ and for some values of Γ; 0.1, 0.2, 0.3 and 0.4 after Ref. [20]. Full curves represent the second-order transition and broken curves the first-order transition. For $\Gamma = 0.4$, the cone region disappears. Vertical line from a to c is an expected temperature-dependence of the Gd state. $\Gamma = 0.2$ for a, 0.3 for b, and 0.4 for c. Full lines denoted by Gd-Y and Gd-La represent the state of the $Gd_{1-x}Y_x$ and $Gd_{1-x}La_x$ alloys at $T = 0$ K, respectively.

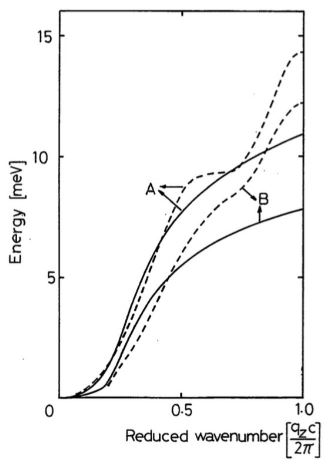

Fig. 15. Magnon dispersions of Gd in the c direction: Curve A at 78 K, and curve B at 232 K; Broken curves by experiment, and full curves by theoretical fitting.