

Quantum Processes and Functional Geometry: New Perspectives in Brain Dynamics

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Abstract. The recent controversy of applicability of quantum formalism to brain dynamics has been critically analyzed. PELLIONISZ and LLINÁS (1982) proposed a functional geometry to understand the internal representation of the events associated to the space-timing of moving objects in the external world. The joint representation of space and time associated to an event as understood by the brain is shown to be different from that understood in modern physics. This indicates that the four-dimensional geometry i.e. Minkowski geometry is not an appropriate description for the internal world. If it is to be the case, the applicability of any kind of quantum field theory in modeling brain function has to be analyzed with great care. Here, the issue of applicability of quantum mechanics to brain function has been discussed in general, from an anatomical perspective and then particular emphasis has been given to the applicability of quantum field theory.

1. Introduction

The possible applicability of quantum formalism, especially in the microscopic level of brain dynamics raises a lot of arguments, and counter arguments indicating the seriousness of the concerned problems among the scientific community (TEGMARK, 2000; HAGAN *et al.*, 2002). Several authors (RICCARDI and UMEZAWA, 1967; STAPP, 1990, 1993; HAMEROFF and PENROSE, 1996; JIBU *et al.*, 1996; VITIELLO, 2001) claimed that quantum processes and collapse of wave function in the brain are of the importance to help us in understanding the information processing and higher order cognitive activities of the brain. Even before, in 1991, PRIBRAM (1991) proposed the holographic model to understand the information processing in the brain.

However, before applying any kind of such approach, the most fundamental, rather thorny issue which should have been resolved, has not been addressed. It is to be noted that before applying any form of quantum mechanics (in the non-relativistic domain), one of the prerequisites is to investigate whether the anatomical structure of the brain permits

assigning any kind of smooth geometric notion like the distance function, or orthogonality in the neuromanifold. For applying quantum field theoretic model to memory function or spontaneous symmetry breaking, one needs to construct space-time geometry in Minkowskian sense over this neuromanifold.

Pelionisz and Llinás (PELIONISZ and LLINÁS, 1982, 1985; LLINÁS, 2002) analyzed the functionality of Central Nervous System (CNS), related to cognition of the event associated to a moving object in the external world. According to their observations, as the conduction speeds through various axons for any external stimulus they are different, and there should exist a time delay at the neuronal level. So the concept of simultaneity (as considered in the special theory of relativity) is hard to be realized conceptually, in case of space-timing for the internal representation of the brain dynamics. We think one should address these issues before applying any kind of quantum formalism to understand the information processing and higher order cognitive activities (ROY and KAFATOS, 2003). Also, it might be interesting to look into the plausibility of finding any form of indeterminacy relation with Planck's constant h or any other kind of constant, say, a **brain constant**, at any level of brain functions.

The intent of this paper is as follows: first we analyze the anatomical structure of the brain and its relation to Euclidean or non-Euclidean distance and then the possibility of assigning space-time (four dimensional) representation. PELIONISZ and LLINÁS (1982, 1985) have shown that our present understanding of brain function does not permit to assign space-time representation. They considered a tensor network theory where they assigned a metric tensor g_{ij} to the Central Nervous System (CNS). However, for global activities of the brain, i.e., to define the metric tensor over the whole neuromanifold, this raises a lot of difficulties. For example, some cortical areas are non-linear or rough, so the tensor network theory becomes very much complicated and almost intractable to solve the mathematical equations. In one of our recent papers (ROY and KAFATOS, 2002), we proposed that the statistical distance function may be considered over the entire neuromanifold considering the selectivity properties of neurons (HUBEL, 1995). In this paper, we explain that the statistical distance function and the statistical metric tensor considered are very important concepts in understanding the above mentioned issues.

2. Functional Geometry and Space-time Representation

The internalization of external geometries into CNS and the reciprocal (PELLIONISZ and LLINÁS, 1985) has created much interest for the last two decades. The central tenet of their hypothesis is that brain is a tensorial system. This hypothesis is based on the consideration of covariant sensory and contra-variant vectors representing motor behavior. Here, CNS acts as the metric tensor which determines the relationship between the contravariant and covariant vectors. The contra-variant observable theorem has been discussed in the context of Minkowskian geometry as well as in stochastic space-time and quantum theory. It can be stated (and can be proved in weak sense) that measurements of dynamical variables are contra-variant components of tensors. This means that whenever a measurement can be reduced to a displacement in a coordinate system, it can be related to contra-variant components of the coordinate system. To make an observation of a dynamical variable as position or momentum, the measurement is usually done in the form

of reading of a meter or similar to that. Through a series of calculations one can reduce the datum to a displacement of in a coordinate system.

MARGENAU (1959) analyzed this issue and claimed that the above reduction can give rise to a measurement if it satisfies the following two requirements. It must be repeatable with the same results and must be physically useful quantity. This can be easily shown in the context of Minkowski space. The motivation of Pellionisz and Llinás was to find a possible single underlying entity, capable of representing any set of particular neuronal networks i.e., whether data derived from particular neuronal networks can be generalized to another set of neuronal networks (from a brain to the brain). This is equivalent to consider the brain as a geometric object. However, they have shown that a space-time representation (in the sense of Minkowski geometry) can not be assigned to the internal representation.

The arguments can be briefly described as follows: CNS function can be compared with a procedure like taking the picture of a moving object not with instantaneous flash but replacing the light with a set of axons (each having a different conduction time). Now, through differently delayed neuronal signals, the simultaneous external events will not be represented in the CNS as **simultaneous**. In other words, *simultaneous onset of firings of a group of neurons with different conduction times will not produce a set of simultaneous external events*. Hence, the assignment of space-time geometry to the functional geometry of neurons is not possible, at least, within the purview of present understanding of brain dynamics.

It appears that *a three dimensional space and one time* can be assigned to the internal representation. But to assign this kind of space and time structure over the global activities of brain (i.e. to account for the holonomic like information processing), one needs to define a smooth metric tensor over the whole neuromanifold. A family of neural networks forms a neuromanifold. However, as some cortical areas of brain are more *non-linear and rough* than others, it seems to be very difficult to construct a smooth metric tensor over the neuromanifold. In fact, the mammalian cerebral cortex has the form of a layered thin shell of gray matter surrounding white matter. The cortical mantle is one of the most important features of the brain and it plays a tremendously important role in understanding brain functions. Although the cortical surface is an important feature of mammalian brain, the precise geometry and variability of the cortical surface are not yet understood clearly. Attempts have been made in the VAN ESSEN LABORATORY to construct mathematical representation of a typical cortical surface. This representation allows us to make statements about the geometry of the surface as well as its variability. Considering the surface as two-dimensional manifold in brain volume, it enables one to compute geometrical properties as the Mean and the Gaussian curvature of the surface.

Recently, NAKAHARA and AMARI (2002) used the concept of *Information Geometry* to understand the geometrical structure of a family of information systems. The information systems consist of a hierarchical structure of neuronal systems with feedback and feed-forward connections. AMARI (2001) introduced a duality structure within the Bayesian framework from the point of view of information geometry. However, he considered a manifold equipped with Riemannian metric formed by a family of distributions. In this framework, the Minkowskian space-time representation, is not realizable due to the lack of existence of positive definite distribution functions for four dimensional space-time.

3. Quantum Formalism

Recent work (STAPP, 1991, 1993; PRIBRAM, 1991; HAMEROFF and PENROSE, 1996; JIBU *et al.*, 1996; BECK, 1996; BECK and ECCLES, 1998; ALFINITO and VITIELLO, 2000) on the applicability of quantum formalism in understanding brain function have led to consider several fundamental issues related to functional geometry of brain.

In the quantum theory of mind-brain, described by Stapp, there are two separate processes. First, there is the unconscious mechanical brain process governed by the Schrodinger equation which involves processing units that are represented by complex patterns of neural activity (or more generally, of brain activity) and subunits within these units that allow “association” i.e., each unit tends to be activated by the activation of several of its subunits. An appropriately described mechanical brain evolves by the dynamical interplay of these associative units. Each quasi-classical element of the ensemble that constitutes the brain creates, on the basis of clues, or cues, coming from various sources, a plan for a possible coherent course of action. Quantum uncertainties entail that a host of different possibilities will emerge. This mechanical phase of the processing already involves some selectivity, because the various input clues contribute either more or less to the emergent brain process according to the degree to which these inputs activate, via associations, the patterns that survive and turn into the plan of action. HAMEROFF and PENROSE (1996) discussed the issue of consciousness taking into consideration quantum coherence. In brief, their model has the following arguments:

1. Quantum coherence and wave function collapse are essential for consciousness and occur in cytoskeletal microtubules and other structures within each of the brain’s neurons.

2. Quantum coherence occurs among tubulins in microtubules, pumped by thermal and biochemical energies. Evidence for some kind of coherent excitation in proteins has recently been reported by VOS *et al.* (1993). The feasibility of quantum coherence in seemingly noisy, chaotic cell environment is supported by the observation that quantum spins from biochemical radical pairs which become separated retain their correlation in the cytoplasm.

3. For the objective reduction as put forward by Penrose and Hameroff, superposed states have each their own space-time geometries. When the degree of coherent mass-energy difference leads to sufficient separation of space-time geometry, the system must choose and decay to a single universe state. Thus Orchestrated Objective Reduction (OOR) involves self selections in fundamental space-time geometry.

4. It is shown that a brain neuron has roughly 10^7 tubulins. If 10% of tubulins within each neuron are involved in quantum coherent state, then roughly 10^3 neurons would be required to sustain coherence for 500 msec, i.e., to reach the quantum gravity threshold for successful occurrence of OOR.

According to these authors, one possible scenario for the emergence of quantum coherence leading to OOR and conscious events is cellular vision. ALBRECHT-BUEHLER (1992) has observed that single cells utilize their cytoskeletons in cellular vision-detection, orientation and directional response to beams of red/infrared light. JIBU *et al.* (1996) argued that this process requires quantum coherence in microtubules and also ordered water.

PRIBRAM (1991) developed the holographic concept in his model of the brain. According to him, the brain encodes information on a three dimensional energy field that enfolds time and space, yet allows to recall or reconstruct specific images from the countless millions of images stored in a space, slightly smaller than a melon. From their experimental findings, he and his collaborators studied in detail the implications of Gabor's quanta of information (GABOR, 1946) for brain's function and their relation to Shannon's measure, on the amount of information from the data. They (JIBU *et al.*, 1996) also studied how the quantum mechanical processes can operate at the synapto-dendritic level. According to their views, something like superconductivity can be operative by virtue of boson condensation over short ranges when the water molecules adjacent to the internal and external hydrophilic layers of the dendritic membrane become aligned by the passive conduction of post synaptic excitatory and inhibitory potential changes initiated at synapses (JIBU *et al.*, 1996).

It is generally argued that the brain is warm and wet. It is interesting to note that recent theoretical and experimental papers support the prevailing opinion (TEGMARK, 2000) that such large warm systems will rapidly lose quantum coherence and classical properties will emerge. In fact, the rapid loss of coherence would naturally be expected to block any crucial role for quantum theory in explaining the interaction between our conscious experiences and the physical activities of our brains. However, HAGAN *et al.* (2002) claimed that at certain level, quantum coherence can be retained as the decoherence time will be effectively delayed.

It is clear from the above discussions that it is necessary to use either non-relativistic quantum mechanics or quantum field theory for the description of activities of the brain. For any kind of quantum field theoretic approach one needs to define the field functions over relativistic space-time as a prerequisite. So far, experimental evidence regarding the anatomy of brain does not permit the space-time description (in a Minkowskian sense), and consequently, any type of quantum field theoretic model is hard to be conceivable, at least, at the present state of understanding in brain dynamics.

However, in order to apply any kind of non-relativistic quantum mechanics, the basic prerequisites are Hilbert space structure and the indeterminacy relation. Before going into the context of indeterminacy relation in brain dynamics, let us try to analyze the micro- and macro- structures in the brain.

3.1. Brain activity at various scales

The different scales of activities of brain can be classified in the following manner (FREEMAN, 1999):

1. Pulses of single neurons, microtubules in milliseconds and microns may be considered as the part of microstructures.
2. Domains of high metabolic demand managed in seconds and centimetres (for measuring the spatial patterns of cerebral blood flow). This can be designated as the macrostructure.
3. Millimeters and tenths of a second are the patterns of the massed dendritic potentials in EEG recordings from the waking and sleeping brains. This can be considered as an area where there might be a level of mesostructure.

FREEMAN (1991) suggested that perception cannot be understood solely by examining properties of individual neurons, i.e., only by using the microscopic approach (dominant approach in neuroscience). Perception depends on the simultaneous activity of millions of neurons spread throughout the cortex. Such global activity is nothing but the macroscopic approach.

Micro- or macrostructures in the brain are distinguished by the scale of time or energy. The macrostructure can be characterized by the fact that the brain lives in hot and wet surroundings with $T = 300$ K. This should be discussed in the context of quantum coherence versus thermal fluctuations. The physiological temperature $T = 300$ K corresponds to an energy $E \sim 1.3 \times 10^{-2}$ eV. Let us now define a signal time $\tau = 2\pi/\omega$ and $\hbar\omega = E$. We then obtain

$$\omega = 2 \times 10^{13} \text{ s}^{-1} \quad \text{or} \quad \tau = 0.3 \text{ ps.}$$

This indicates that the physiological temperatures correspond to frequencies **smaller** than the *picosecond* scale. They correspond to the time scale involving electronic transitions like electron transfer or changes in molecular bonds. In cellular dynamics, the relevant time scale is of the order of $\tau > 0.4$ ns, where, $E_{\text{cell}} \sim 10^{-5}$ eV (BECK, 1996). To allow comparison with quantum scales, let us distinguish the two scales as follows:

1. The macroscopic or cellular dynamics level with time scales in the *milli*, down to the *nanosecond* range.
2. The microscopic or quantal dynamics level with time scales in the *pico*, down to the *femtosecond* range.

The large difference between the two time scales indicates quantum processes might be involved in the individual microsites and decoupled from the neural networks. Recently, BERNROIDER (2003) made an analysis by applying the concept of Lagrangian action to brain processes at different scales of resolutions in order to clarify the current dispute whether classical neurophysics or quantum physics are relevant to brain function. The central issue of his analysis is to estimate the order of action from dimensional analysis, relevant to brain physiology and its close proximity to quantum action i.e., \hbar (Planck's constant). For example, spiking action at the single cell level is found to involve 1.8×10^{-15} Lagrange (using mechanical units) down to 2.1×10^{-16} . This lies between 10^{18} and 10^{19} times Planck's constant which is in good agreement with the time scale difference and spiking time estimated by TEGMARK (2000). However, he pointed out that the action behind the selective ion permeation and channel gating might be of interest at the molecular level. Considering 10^5 ions permeating per msec and employing a saturating barrier model with one or two ions crossing during that time, the action turns out to be of the order of 0.48×10^{-34} Lagrange, which is in the range of the quantum action \hbar (1.05459×10^{-34} MKS units). It implies that brain functioning at a certain level might be a proper arena to apply the quantum formalism.

3.2. Indeterminacy relations

In communication theory, GABOR (1946) considered an uncertainty relation between frequency (ω) and time (t) as

$$\delta\omega\delta t = \frac{1}{2}.$$

This is similar to the Heisenberg energy (E)-time (t) uncertainty relation:

$$\delta E \delta t = \hbar \left[\hbar = \frac{h}{2\pi} \right]$$

where h is Planck's constant. Now, if the quantum formalism is considered to be valid (even in its non-relativistic form) in brain dynamics, there should exist a similar type of uncertainty relation between frequency/energy and time, i.e.,

$$\delta\omega\delta t = b$$

where b may be termed here as the “*brain constant*”. Even if there exists such a constant in brain dynamics, one needs to relate it to action quanta like Planck's constant. *Future research in brain function might shed light on this important issue.*

As far as the existence of Hilbert space structure is concerned, one needs to define a smooth distance function over the cortical surface of the brain. It should be mentioned that JOLIOT *et al.* (1994) found a minimum interval in sensory discrimination. Considering this aspect, they claimed that consciousness is a non continuous event determined by the activity in the thalamocortical system. So one needs to introduce discrete time or granularity in space and time geometry.

4. Probabilistic Geometry and the Neuromanifold

We describe first the geometroneuro-dynamics as proposed by ROY and KAFATOS (2002) considering the neurophysiological point of view. Then we shall proceed to the generalization of this approach and investigate the relevant issues from a more generalized perspectives.

4.1. Orientation selectivity of neurons and statistical distance

Recent research on Planck scale physics (ROY, 2003) sheds new light on the possible geometrical structure for discrete and continuum levels. The idea of probability in geometric structure as proposed and developed by MENDER (1942, 1949), seems to be a very useful tool in defining distance function over the cortical areas of brain.

There is a large variety as well as number of neurons in the brain. In such case, collective effects which can only be accounted for in terms of statistical considerations, are clearly important. Experimental evidences point to more than 100 different type of neurons in the brain, although the exact number is not yet decided. It is found that no two neurons are identical, and it becomes very difficult to say whether any particular difference represents more than the other i.e., between individuals or between different classes. Neurons are often organized in clusters containing the same type of cell. The brain contains

thousands of clusters of cell structures which may take the form of irregular clusters or of layered plates. One such example is the cerebral cortex which forms a plate of cells with a thickness of a few millimeters.

In the visual cortex itself (HUBEL, 1995), certain clear, unambiguous patterns in the arrangement of cells with particular responses have been found. Even though our approach could apply to non-visual neurons, here we limit our study to the neurons in the visual cortex as the visual cortex is smoother, preventing non-linear effects. For example, as the measurement electrode is moving at right angles to the surface through the grey matter, cells encountered one after the other have the same orientation as their receptive field axis. It is also found that if the electrode penetrates at an angle, the axis of the receptive field orientation would slowly change as the tip of the electrode is moved through the cortex. From a large series of experiments in cats and monkeys it was found:

Neurons with similar receptive field axis orientation are located on top of each other in discrete columns, while we have a continuous change of the receptive field axis orientation as we move into adjacent columns.

The visual cortex can be divided into several areas. The most important areas in the visual cortex are V1, V2, V3, V4 and MT (V5). The primary visual cortex area V1 is important for vision. It is the principal entry point for visual input to the cerebral cortex. From this area, relays pass through a series of visual association areas in parietal and temporal regions and finally to the prefrontal cortex where substrates for decision making on the basis of visual cues are found. The main issues related to visual cortex are linked to intrinsic and extrinsic relays of each cortical region, geometrically ordered microcircuitry within appropriate areas etc. Because of the stripy appearance of area V1, this area is also known as the striate and other areas as the extrastriate (nonstriate) visual cortex. For example, in the monkey striate cortex, about 70% to 80% of cells have the property of orientation specificity. In a cat, all cortical cells seem to be orientation selective, even those with direct geniculate input (HUBEL, 1995). Hubel and Wiesel found a striking difference among orientation-specific cells, not just in the optimum stimulus orientation or in the position of the receptive field on the retina, but also in the way cells behave. The most useful distinction is between two classes of cells: simple and complex. These two types differ in the complexity of their behavior and one can make the reasonable assumption that the cells with the simpler behavior are closer in the circuit to the input of the cortex.

The first oriented cell recorded by HUBEL (1995) which responded to the edge of the glass slide was a complex cell. The complex cells seem to have larger receptive fields than simple cells, although the size varies. Both type of cells do respond to the orientation specificity. There are certain other cells which respond not only to the orientation and to the direction of movement of the stimulus but also to the particular features such as length, width, angles etc. They originally characterized these as hypercomplex cells but it is not yet clear whether they constitute a separate class or, represent a spectrum of more or less complicated receptive fields. Based on this observations, we may ask now, how the computational structure or filters can manifest as orientation detectors?

To have the answer of this question whether single neurons serve as feature or channel detectors, in fact, PRIBRAM and his collaborators (1981 and references therein) made various attempts to classify "cells" in the visual cortex. This proved to be impossible because each cortical cell responded to several features of the input such as orientation,

velocity, the spatial and temporal frequency of the drifted gratings. Furthermore, cells and cell groups displayed different conjunctions of selectivities. From these findings and analysis, Pribram and his group concluded that cells are not detectors, that their receptive field properties could be specified, but rather the cells are multidimensional in their characteristics (PRIBRAM, 1991). Thus, the pattern generated by an ensemble of neurons is required to encode any specific feature, as found by Vernon Mountcastle's work on the parietal cortex and Georgopoulos's data (PRIBRAM, 1998) on the motor cortex.

Again, it is worth mentioning that when discussing perception, FREEMAN and his collaborators (1991) suggested that perception cannot be understood solely by examining properties of individual neurons i.e. by using the microscopic approach that currently dominates neuroscience research. They claimed that perception depends on the simultaneous, cooperative activities of millions of neurons, spread throughout the expanses of the cortex. Such global activity can be identified, measured and explained only if one adopts a macroscopic view alongside the microscopic one.

4.2. *Statistical distance*

We can define the notion of distance between the “filters” or the orientation selective neurons which is similar to the statistical distance between quantum preparations as introduced by WOOTTERS (1981). The notion of statistical distance can most easily be understood in terms of photons and polarizing filters:

Let us consider a beam of photons prepared by a polarizing filter and analyzed by a nicol prism and $\theta \in [0, \pi]$ be the angle by which the filter has been rotated around the axis of the beam, starting from a standard position ($\theta = 0$) referring to the filter's preferred axis as being vertical. Each photon, when it encounters the nicol prism, has exactly two options: to pass straight through the prism (with “yes” outcome) or to be deflected in a specific direction characteristic of the prism (“no” outcome). We assume that the orientation of the nicol prism is fixed once and for all in such a way that vertically polarized photons always pass straight through. By counting how many photons yield each of the two possible outcomes, an experimenter can learn something about the value of θ via the formula $p = \cos^2\theta$, where p is the probability of “yes” (WOOTTERS, 1981), as given by quantum theory.

If we follow this analogy in the case of oriented neurons in the brain i.e., as if the filters are oriented in different directions like oriented analyzers, we can proceed to define the statistical distance.

4.3. *Statistical distance and Hilbert space*

WOOTTERS (1981) first showed that the statistical distance between two preparations is equal to the angle in Hilbert space between the corresponding rays. The main idea can be explained as follows, imagining the following experimental set up:

Let there be two preparing devices, one of which prepares the system in a specific state, say ψ^1 , and the other prepares in ψ^2 . Here, the statistical distance between these two states can be thought of as the measure of the number of distinguishable preparations between ψ^1 and ψ^2 . However, in treating quantum systems, new features should be observed as opposed to rolling the dice. For a dice, there is only one possible experiment to perform, i.e., rolling the dice, whereas for a quantum system, possibility is many, one for each different analyzing device. Furthermore, two preparations may be more easily

distinguished with one analyzing device than with another. For example, the vertical and horizontal polarizations of photons can easily be distinguished with an appropriately oriented nicol prism, but can not be distinguished at all with a device whose eigenstates are the right and left handed circular polarizations. Due to this reason, one can speak of the statistical distance between two preparations ψ^1 and ψ^2 as related to a particular measuring device which means the statistical distance is device dependent. The absolute statistical distance between ψ^1 and ψ^2 is then defined as the largest possible such distance, i.e., statistical distance between ψ^1 and ψ^2 when the states are analyzed by the most appropriate or discriminating apparatus. We can illustrate this point little more in the following way:

Let $\phi_1, \phi_2 \dots \phi_N$ be the eigenstates of a measuring device A , by which ψ^1 and ψ^2 are to be distinguished. It is assumed that these eigenstates are non-degenerate so that there are N -distinct outcomes of each measurement. The probabilities of various outcomes are $|\langle \phi_i, \psi^1 \rangle|^2$ if the apparatus is described by ψ^1 and $|\langle \phi_i, \psi^2 \rangle|^2$ if the apparatus is described by ψ^2 . Then the statistical distance between ψ^1 and ψ^2 with respect to the analyzing device A is

$$d_A(\psi^1, \psi^2) = \cos^{-1} \left[\sum_{i=1}^N |\langle \phi_i, \psi^1 \rangle| |\langle \phi_i, \psi^2 \rangle| \right]. \quad (1)$$

This quantity attains its maximum value if it assumes one of the eigenstates of A , (say, ϕ_1). In that case, we get the statistical distance as

$$d(\phi^1, \phi^2) = \cos^{-1} |\langle \phi^1, \phi^2 \rangle|. \quad (2)$$

This clearly indicates that the statistical distance between two preparations is equal to the angle in Hilbert space between the corresponding rays. The equivalence between the statistical distance and the Hilbert space distance might be very surprising at first. It gives rise to the interesting possibility that statistical fluctuations in the outcome of measurements might be partly responsible for Hilbert space structure of quantum mechanics. These statistical fluctuations are as basic as the fact that quantum measurements are probabilistic in their nature.

However, it should be mentioned that although representation of orientation of objects in the visual cortex is fairly fine-scaled, visual information regarding the non striate visual processing and in superior colliculus is very rough and varies in a non-linear way from that in striate cortex. This type of nonlinearity is neglected here as we have considered statistical considerations which average out this type of nonlinearity. Instead, we considered here the distance between the different clusters of neurons or between the ensemble of neurons.

4.4. Perception and relational aspects in probabilistic geometry

The issue of continuum and discreteness remains a long standing problem over the last few centuries. In mathematics, if the quantity A is equal to the quantity B and B is equal to C , then A is equal to C , i.e., mathematical equality is a transitive relation. In the observable continuum "equal" means indistinguishable. In psychology, following FECHNER (1860),

we can say that A may lie within the threshold of B and B within the threshold of C . POINCARÉ (1905) suggested “for the raw result of experience, $A = B, B = C; A \leq C$ which may be regarded as the formula for the physical continuum. MENGER (1949) tried to solve this problem from the positivist point of view. Following his words:

“Instead of distinguishing between a transitive mathematical and intransitive physical relation, it thus seems more hopeful to retain the transitive relation in mathematics and to introduce for the distinction of physical and psychological quantities a probability, that is, a number lying between 0 and 1”.

He considered the role of probability in geometry and introduced the concept of probabilistic metric as well as the concept of a set of hazy lumps instead of considering set of points. Then the problem turns out to be similar to finding a probability of the overlapping lumps. For more intuitive understanding, the lumps were considered as the “seat” of elementary particles like electrons, protons etc. These lumps are taken as not to be reducible to any other structures. In other words, they are the ultimate building blocks of space and time. Therefore, a kind of *granularity* is introduced here at the very basic level. Mathematically speaking, it can be stated as:

“for each pair of elements A and B of probabilistic geometry, it is possible to associate a distribution function $F_{AB}(z)$ which can be interpreted as the probability that the distance between the points is less than z ”.

Essentially the relational aspect of geometry has been proposed and elaborated by Menger. He replaced the usual metric function by a distribution function and showed that this distribution function satisfies all the axioms of the metric. Hence it is known as **probabilistic metric space**. There are various types of probabilistic metric spaces used in different branches of physical science (SCHWEIZER and SKLAR, 1983; ROY, 1998). On a large scale, taking averages over these distributions, one can get the usual metric structure. Recently, MOGI (1997) tried to reinterpret Mach’s principle in the context of the response selectivity of neurons. He proposed that in perception, the significance of firing of a neuron is determined by the relation of the firing to other neurons at that very psychological moment. He called it *Mach’s principle in perception*. According to his proposal, it is not meaningful to talk about the firing of a single neuron in isolation and its significance in perception.

In our approach, toward geometro-neurodynamics (ROY and KAFATOS, 2004), we have considered the same relational aspect of geometry by considering the orientation selectivity of neurons. Here, we consider the stochastic space as proposed by FREDERICK (1976). In this model, the actual points of the space are stochastic in nature which can not be used as either a basis for a coordinate system or to define a derivative. However, the space of common experience at large scales or in the laboratory frame is non stochastic. Therefore, we start from large scale non stochastic space and then continue to build up a structure by applying it mathematically toward stochastic space i.e. toward small scales. This stochasticity is considered to be manifested in a stochastic metric tensor g_{ij} and the corresponding mass distribution determines not only the space geometry but also the space

stochasticity. However, as more and more mass is confined in a region of space, the less stochastic will be that space. Let us start with the relation between a covariant and contravariant quantity, i.e.,

$$x^i = g^{ij}x_j,$$

As g_{ij} is stochastic, one obtains a distribution of the contravariant quantity x^i instead of a fixed quantity. Now, assuming a Lagrangian, taking this kind of stochastic metric, we define a pair of conjugate variables, one covariant and another contravariant, as

$$P_j = \frac{\partial L}{\partial \dot{q}^j}$$

where P_j is a covariant quantity and is not observable in the laboratory due to its covariant nature. The observable quantity is the contravariant one, i.e.,

$$P^j = g^{jv} P_v.$$

As the metric tensor g^{ij} is of stochastic in nature, P^j is a random variable. So, if one member of the conjugate pair is well defined, the other member will be random. This may play a significant role in accounting for the tremor in motor behavior in neurophysiological experiments.

4.5. Neurophysiological basis for the stochasticity in the metric

Let us now look into the origin of stochasticity in neuromanifold. The neurophysiological evidence shows that most neurons are spontaneously active, spiking at random intervals in the absence of input. Different neuron types have different characteristic spontaneous rates, ranging from a few spikes per second to about 50 spikes per second. The mechanism of regular activity is well studied whereas the mechanism of random spontaneous activity is not well understood. Several possibilities are discussed by LINDHAL and ARHEM (1994).

One is the well known ion-channel hypothesis. According to this, the nerve impulses are triggered by the opening of single ion channels where the ion channel gating is random. Ion channels are membrane proteins through which the current causing the nerve impulse passes. DONALD (1990) considered that the randomness may be related to quantum fluctuations. LINDHAL and ARHEM (1994) suggested that single channels may cause spontaneous activity in areas of the brain with consciousness. However, the detailed mechanism of ion-channel gating is still not fully understood. GRANDPIERRE (1995) made an attempt to study the effect of the fluctuation of the zero point field (ZPF) in the activity of brain. As such, the future investigations on the effect of ZPF on the neurons may shed new insight not only for the spontaneous activity of neurons but also on the actual process of consciousness. We like to emphasize that in our picture, the fluctuation associated with this kind of spontaneous activity of neurons is the physical cause behind the stochasticity of the metric tensor.

4.6. Frederick's approach to the stochastic metric

To start with, let us take FREDERICK's (1976) version of stochasticity in geometry. He made several interesting postulates as follows:

1. The metric probability postulate: $P(x, t) = A \sqrt{(-g)}$, where for a one-particle system $P(x, t)$ is the particle probability distribution, A is a real valued function, and g is the determinant of the metric.

2. **The metric superposition postulate:** If at a position of a particle the metric due to a specific physical situation is g_{ij}^1 and the metric due to a different physical situation is g_{ij}^2 , then the metric at the position of the particle due to the presence of both of the physical situations is g_{ij}^3 can be written as $g_{ij}^3 = (1/2)[g_{ij}^1 + g_{ij}^2]$.

3. **The metric ψ postulate:** There exists a local complex diagonal coordinate system in which a component of the metric is at the location of the particle described by the wave function ψ .

We have started with Frederick's approach not only for the attractive mathematical framework for neuromanifold but also for the use of Mach's principle as the guiding rule for stochastic geometry. It then becomes possible to derive quantum mechanics by adopting a strong version of Mach's principle such that in the absence of mass, space becomes non-flat and stochastic in nature. At this stage, the stochastic metric assumption is sufficient to generate the spread of wave function in empty space. Following this framework, one obtains an uncertainty product for contravariant position vector (q^1) and contravariant momentum vector (P^1) as

$$\Delta q^1 \Delta P^1 = \Delta q^1 \Delta (P_\nu g^{\nu 1})$$

where $P^j = g^{j\nu} P_\nu$. Now the question is what is the minimum value of this product. It can be shown that

$$\Delta q^1 \Delta (P_{\text{vmin}} g^{\nu 1}) = \Delta q^1 \Delta P^1 > k_{\text{min}}$$

which is nothing but the uncertainty principle with k as the action quantity similar to h , i.e., Planck's constant. Moreover, using the superposition postulate of the metric tensor, it is also possible to account for the interference phenomena. But the problem is that it is not properly understood at what level of the brain activities, the quantum effects will be prevalent. The understanding of ion channel activity may provide the answer for the above questions.

4.7. Penrose and Hameroff approach: OrchOR model

It is to be noted that the above metric superposition postulate can be shown to be valid under the weak approximation of the general theory of relativity. If there is more non-linearity in the cortical surface, the superposition may actually break down. In Hameroff and Penrose model, they considered a kind of superposition of space and time geometries in order to relate it with the superposition of wave functions and the decoherence due to variation of mass distribution and hence due to gravity effects. In the above framework, it is possible to relate the superposition of wave functions using the superposition postulate

of metrics. Because of the existence of different curvatures at different points (due to different mass distributions) in the framework of statistical geometry, one can write the superposition of metrics or geometries. But it is necessary to investigate the real neurophysiological conditions under which the superposition of metrics would be a valid approximation. It raises a new possibility of constructing a Hilbert structure over the neuromanifold within the framework of statistical geometry. However, as we mentioned earlier, construction of Hilbert structure over the neuromanifold is one of the prerequisites before applying any kind of quantum mechanical process to the brain.

5. Information Processing in the Brain

The information generated by integrated neural processes and its measurement has created a lot of interests among the scientific community for the last few years. The measure of information essentially depends on the basis of statistical foundation of information theory (SHANNON, 1948). One of the intriguing question arises is how far the statistical aspects of information theory can help one to assign a measure to differentiate the informative character of the neural processes without any reference to an external observer. The issue of the external observer has been debated in various branches of science and philosophy over the last century since the birth of quantum mechanics. In fact, the issue of measurement procedure in the history of science has been reanalyzed from time to time but still under active considerations after the mathematical formulation of Von Neumann, using the statistical concept of entropy. In the standard approach, one generally assigns a number to measure the information and probability of the states of the system that are distinguishable from the point of view of an external observer. But, the brain not only processes the information, it also interprets the pattern of activities (PRIBRAM, 1991). Therefore, one must avoid the concept of a privileged viewpoint of an external observer to understand the information processing in the neural processes in the brain.

In our approach, we have developed a framework (ROY *et al.*, 2003) where it is possible to avoid the concept of an external observer by reanalyzing the very basis of measurement procedure as well as the neurophysiological evidences in the standard paradigm. EDELMAN *et al.* (2000) discussed this problem in the context of neurophysiology and consciousness. The main problem is how to measure the differences within a system like the brain? He defined a concept of mutual information for this purpose. There, the authors considered the entropic measure to define the information as considered by Shannon.

The principal idea lying behind our approach can be summarized as follows: The concept of invariance plays a crucial role to understand the *information processing* and *measurement issues* in the brain. In the brain, a matching occurs between an input pattern and a pattern inherent in the synaptodendritic network by virtue of generic or learning experience. In the Holonomic theory, both the input and output patterns provide the initial conditions. The match between them is considered to be probabilistic in nature (PRIBRAM, 1991). We have introduced here a kind of invariance assisted by the context (as described by the inherent patterns in the dendrites). This is quite similar to environment-assisted invariance in Quantum Mechanics (ZUREK, 2000). It is one of the fundamental principles of quantum mechanics known as *quantum determinism* where it is possible to show that an

entangled state is formed between the input and output pattern. This state can be written in terms of its basis vectors. Now, picking up the specific term from the expansion is generally known as selection. REDHEAD (1989) emphasized that the selection of the parts is related to the attention to a particular sub-ensemble of the whole. This means selection is not part of quantum physics.

In physicist's language, the selection signifies measurement that marks the end of quantum physics. In contrast, the "yes-no"-experiment puts the selection process at the beginning and makes the involvement of brain dynamics (or the selection that underlies the pattern recognition in the brain) into the primitive of quantum mechanics. It may be mentioned that "yes-no"-experiment depends primarily on the act of cognition. In this framework it has been shown how the above kind of analysis and the concept of invariance will help us to understand the nature of ignorance (for example to understand the probabilistic nature of matching) and hence the origin of probability in the context of brain function, similar to quantum physics, without using concepts like *collapse* or *measurement* as commonly used in quantum mechanics. It is curious to note that EDELMAN *et al.* (2000) pointed out that selection is biologically the more fundamental process. He conjectured that there exist two fundamental ways of patterning thought: *selectionism and logic*. We think that the selectionism plays a very significant role in understanding information processing in brain.

6. Conclusions

The above analysis clearly indicates that it is not understandable how the anatomy of brain can permit the joint space-time representation in the sense of the special theory of relativity. Therefore, the applicability of any kind of quantum field theoretic approach is not realizable, at least, at the present stage of understanding of brain function. However, it may be possible to define a smooth distance function and metric tensor in the probabilistic sense. The probabilistic geometry seems to play a significant role in understanding Hilbert space structure and its connection to non-relativistic quantum mechanics. This approach sheds new light to understand the information processing and measurement procedure related to the brain. The implication of stochastic geometry in the inner world might have significant effects in the external world too, which will be considered in subsequent publications.

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