# Generation of Mandala Patterns from Texts that Include Sutras, Poems and Strings of Words: Method and Examples 

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#### Abstract

Two-dimensional patterns with four-fold mirror symmetry are generated from texts written with Chinese characters. The drawing is based on the spiral mapping technique, with which one can visualize a statistical property of a text within a twodimensional region through spirally folding a string of the characters. After illustrating the outline of the method, typical results of the pattern generation are shown and subsequently analyzed by means of the diversity indices. Specifically, these patterns can be obtained from a variety of texts such as sutras, poems, and strings of words, for instance, titles of chapters in a literary work.


## 1. Introduction

Generation, formation, and evaluation of patterns have been of major research topics in science of form (TAKAKI, 2003). Traditionally important patterns include Arabesque, spiral, lattice, flowing water, and foreign patterns. In addition to these, there are a quantity of refined and sophisticated patterns in the world. One could see them, for instance, in Oriental (Persian) carpets (BIER, 2000), rugs, crochet laces, chinawares, Japanese family crests, Ainu embroideries, and textiles in a barbarous society. In religious contexts, patterns have been realized as mandalas in the Esoteric Buddhism, rosettes in churches, and decorative arts with Arabic characters. Of these, mandalas are highly symmetrical arrangements of sacred symbols, which constellate around the center (TUCCI, 1969; MATSUNAGA, 1991). Originally, they were used as means of the religious achievement in the Hinduism as well as the Buddhism. On the basis of this initial meaning, recent progress in the depth psychology has found relevance between the highly symmetrical drawings and the traditional mandalas. JUNG (1968) had found through observations of his patients that such singular figures with rigorous symmetries could be interpreted with a symbol of the 'Self' being a key concept in his psychology (JACOBI, 1939). In his point of view, it is the mandala that appears as a symbol of 'Self' which attemps to integrate various conflicting elements in the mind. In his works, he classified mandalas into the two categories: Type I)
'modern or personal mandalas' as a natural symbol, and Type II) 'traditional or historical mandalas' as a cultural symbol. According to this classification, the mandalas used in the Esoteric Buddhism can be included in the latter. Based on the Jung's understanding, more recently TUCCI (1969) has defined mandalas as 'figures symbolically representing the two processes that consist of break-up and reintegration of the consciousness.'

In this paper two-dimensional patterns with four-fold mirror symmetry are generated from texts written with Chinese characters. The drawing is based on the spiral mapping technique (HAYATA, 2003), with which one can visualize a statistical property of a text in a two-dimensional region through spirally folding a string of the characters. With the Jung's classification mentioned above, these symmetrical patterns can be categorized into Type-I mandalas. Indeed, in his collected works concerning mandalas (JUNG, 1968), he presented a variety of patterns with four-fold mirror symmetry. (See, e.g., Figs. 5-9 in the literature.) After describing the outline of the method, typical results of the pattern generation are shown and subsequently analyzed by means of the diversity indices. Specifically, the author employs the Simpson's index of diversity and the Shannon's information entropy (the average amount of information). Mandala patterns are obtainable from a variety of texts such as the HANNYA SHINGYO, a figure in the KEGON Scriptures, the RISHU Sutra, a Chinese poem, and strings of words, such as, for instance, titles of chapters in a literary work. Here the HANNYA SHINGYO is an extremely condensed form of a great scripture termed the HANNYA Sutra; the KEGON Scriptures, which are regarded as being representative of the Mahayana Scriptures, have commonly been used in the KEGON sect, while the RISHU Sutra has been used in the Shingon sect. Words are cited from the Tale of Genji, Han Fei Tzu (KAMPISHI), SHOBOGENZO, KIGOes in haiku, and eventually the 37 Sacred Symbols in the KONGO World. The explanation of these texts is given in the caption of each pattern. Comparison is made among features of the mandalas.

## 2. Procedure for Mandala Generation

In this section, a method for generating mandala patterns is first described through application to the Sexagenary Cycle (see, e.g., OKADA, 1999) shown in Fig. 1. Subsequently, typical examples of mandala generation are presented.

### 2.1. Mapping procedure

In the present method, we take notice of fluctuation in the number of kanas along the sequence of words written with Chinese characters. Note that kanas are letters in the Japanese syllabary, which are frequently added to a Chinese character in order to indicate its pronounciation. For instance, for No. 31 on Fig. 1, the two Chinese characters composing the 31st word are accompanied by two kanas [2] and another [1], respectively. Here the numeral in the bracket represents the number of kanas per Chinese character, which can be used as an indicator for the 'length' of each Chinese character. In Fig. 1, the length data are defined clearly; for No. 31 those are $2 \cdot 1$.

To visualize the sequence of the character-length data, the author presents a method that modifies the spiral mapping technique (HAYATA, 2003). In this method, from a point on the outermost orbit to the center $(0,0)$, a spiral pattern with the clockwise rotation is

| 《六十干支》 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1）甲子 | 加つ | 2•1 | 31）甲午 | こうご | 2－1 |
| 2）乙丑 | いつ 5 ¢ ${ }^{\text {j }}$ | $2 \cdot 3$ | 32）乙未 | いう | $2 \cdot 1$ |
| 3）丙寅 | べしん | $2 \cdot 2$ | 33）丙申 | ヘ兄しん | $2 \cdot 2$ |
| 4）丁卯 | てい住 | $2 \cdot 2$ | 34）丁酉 | てい ${ }^{\text {而 }}$ | $2 \cdot 2$ |
| 5）戊辰 | 园しん | $1 \cdot 2$ | 35）戊戌 | （どゅ | $1 \cdot 3$ |
| 6）己巳 | きし | $1 \cdot 1$ | 36）已亥 | きが䊽 | $1 \cdot 2$ |
| 7）庚午 | こうご | $2 \cdot 1$ | 37）庚子 | こうし | 2－1 |
| 8）辛末 | しん ひ | $2 \cdot 1$ | 38）辛丑 | しん ちゅう | $2 \cdot 3$ |
| 9）壬申 | じん しん | $2 \cdot 2$ | 39）壬寅 | c゙h Uh | $2 \cdot 2$ |
| 10）癸酉 | き ${ }^{\text {d }}$ | $1 \cdot 2$ | 40）癸卯 | き臨 | $1 \cdot 2$ |
| 11）甲戌 | こうじゅつ | $2 \cdot 3$ | 41）甲辰 | こう しん | $2 \cdot 2$ |
| 12）乙亥 | 以加隹 | $2 \cdot 2$ | 42）乙巳 | 以 | 2－1 |
| 13）丙子 | べし | $2 \cdot 1$ | 43）丙午 | ヘ号ご | 2． 1 |
| 14）丁丑 | てい 5 ¢う | $2 \cdot 3$ | 44）丁未 | てい $\downarrow$ | 2－1 |
| 15）戊寅 | 生いん | $1 \cdot 2$ | 45）戊申 | 咸しん | $1 \cdot 2$ |
| 16）已卯 | き臨 | $1 \cdot 2$ | 46）己酉 | き ${ }^{\text {¢ }}$ | $1 \cdot 2$ |
| 17）康辰 | こう しん | 2－2 | 47）庚戌 | こうじゅ | $2 \cdot 3$ |
| 18）辛巳 | しんし | 2－1 | 48）辛亥 |  | 2－2 |
| 19）壬午 | ぜん ご | 2－1 | 49）壬子 | どんし | 2－1 |
| 20）癸末 | き ひ | $1 \cdot 1$ | 50）癸丑 | きちゅう | 1－3 |
| 21）甲申 | こうしん | $2 \cdot 2$ | 51）甲寅 | こう Wh | 2－2 |
| 22）乙西 | 以 ${ }^{\text {a }}$ | $2 \cdot 2$ | 52）乙卯 | 的時 | 2－2 |
| 23）丙戌 | へいじゅつ | 2•3 | 53）丙辰 | an しん | 2－2 |
| 24）丁亥 | ていが兄 | $2 \cdot 2$ | 54）丁尸 | ていし | 2－1 |
| 25）戊子 | 俥こ | $1 \cdot 1$ | 55）戊午 | 俥ご | 1－1 |
| 26）己\＃ | き ¢ $_{\text {¢ }}$ | 1－3 | 56）己未 | き ひ | 1－1 |
| 27）庚寅 | こう Wh | 2－2 | 57）庚申 | こう しん | $2 \cdot 2$ |
| 28）辛卯 | しった㖘 | $2 \cdot 2$ | 58）辛酉 | しん しん | 2．2 |
| 29）壬辰 | じん しん | 2－2 | 59）壬戌 | じんじゅ | $2 \cdot 3$ |
| 30）癸巳 | きし | $1 \cdot 1$ | 60）癸亥 | き加い | $1 \cdot 2$ |

Fig．1．The Sexagenary Cycle．
drawn in accordance with the direction of a sequence．However，because of the uncertainty in the location of the initial point，in actual drawings，instead of the forward propagation， a spiral with the counterclockwise rotation is created backward（i．e．，from No． 60 to No．1）． The reverse trajectory is generated as the following algorithm：

1）Starting from the center $(0,0)$ ，move horizontally along the $x$－axis with the increment $\Delta x$ ．Here $\Delta x(>0)$ is the length of the last Chinese character of the sequence．For the system of Fig．1，$\Delta x=2$ ．

2）Subsequently，move upwards with the increment $\Delta y$ ．Here $\Delta y(>0)$ is the length of the second Chinese character from the terminal．For Fig．1，$\Delta y=1$ ．

3）For the point being in the first section $(x>0, y>0)$ ，move backward along the horizontal direction（i．e．，set $\Delta x<0$ ，being the decrement）and upwards along the vertical direction（set $\Delta y>0$ ）until the point attains into the second section $(x<0, y>0)$ ．

4）For the point being in the second section，move backward along the horizontal direction（set $\Delta x<0$ ）and downward along the vertical direction（set $\Delta y<0$ ）until the point attains into the third section $(x<0, y<0)$ ．


Fig. 2. Spirally mapped orbits for the Sexagenary Cycle shown in Fig. 1.
5) For the point being in the third section, move forward along the horizontal direction (set $\Delta x>0$ ) and downward along the vertical direction (set $\Delta y<0$ ) until the point attains into the fourth section $(x>0, y<0)$.
6) For the point being in the fourth section, move forward along the horizontal direction (set $\Delta x>0$ ) and upwards along the vertical direction (set $\Delta y>0$ ) until the point attains into the first section $(x>0, y>0)$.
7) Go to Step 3 and repeat this procedure until the point attains the initial Chinese character.

Applying this algorithm to the sequence of Fig. 1, one obtains the trajectory

$$
\begin{align*}
& (0,0) \rightarrow(2,0) \rightarrow(2,1) \rightarrow(-1,1) \rightarrow(-1,-1) \rightarrow(1,-1) \rightarrow(1,1) \\
& \rightarrow(-1,1) \rightarrow(-1,-1) \rightarrow(0,-1) \rightarrow(0,-2) \rightarrow(1,-2) \rightarrow(1,-1) \rightarrow(2,-1) \\
& \rightarrow(2,1) \rightarrow(0,1) \rightarrow(0,3) \rightarrow(-2,3) \rightarrow(-2,1) \rightarrow(-4,1) \rightarrow(-4,-1) \\
& \rightarrow \cdots \\
& \rightarrow(-4,16) \rightarrow(-4,14) \rightarrow(-7,14) \rightarrow(-7,12) \rightarrow(-8,12) \rightarrow(-8,10) \tag{1}
\end{align*}
$$

Here the adjacent points are joined with a segment line. The spiral pattern is thus obtained as shown in Fig. 2, which becomes a basis for reproducing copies. Specifically, we consider three derivatives arising from the original pattern shown in Fig. 2: 1) the map reversed across the $y$-axis, 2) the map reversed across the $x$-axis, and 3 ) the map reversed across the center $(x, y)=(0,0)$. Subsequently, in order to obtain a pattern with two-fold mirror


Fig. 3. The mandala pattern generated from Fig. 2, where $(S, f, f / N[\%])=(1,196,71.0),(2,48,17.4),(4,16$, $5.8),(12,4,1.4),(16,4,1.4),(31,4,1.4)$, and $(35,4,1.4)$ with $N=276(m=7)$. Here $N(m)$ stands for the number of polygonal elements (the number of their category).
symmetry, these four maps are superimposed without offset. Finally, this pattern and its $90^{\circ}$-rotation are superimposed. Note that the final pattern maintains the four-fold mirror symmetry being a typical feature of mandalas. The final result can be seen in Fig. 3. It should be mentioned here that more complicate and/or dynamic patterns could be generated by utilizing longer texts such as, e.g., sutras or dharanies in the Buddhism and Chinese poems, which will be described in the subsequent section. In other words, the degree of complexity would be controllable by selecting an appropriate text.

### 2.2. Examples

In this section typical results of mandala patterns generated from texts are displayed. The texts written with Chinese characters include sutras, a poem, and sequences of words. As the most famous sutra in the Buddhism, first, consider the HANNYA SHINGYO (see, e.g., Hiro, 1998) as shown in Fig. 4(a), from which the mandala pattern of Fig. 4(b) has been obtained. The frequency $(f)$ distribution of the elemental area $S$, i.e., $\{S\}$ vs. $\{f\}$, is given in the caption of Fig. 4(b). Note here that the pattern is composed of 556 polygonal elements (i.e., $N=556$ ).

Subsequently, we consider the ICHIJO-HOKKAI figure in the KEGON Scriptures (Kamata, 1988). As seen in Fig. 5(a), this figure can be featured by a meandering string of 210 Chinese characters. To make mapping the original geometry has been rewritten as shown in Fig. 5(b). The pattern created from Fig. 5(b) is seen in Fig. 5(c), where the
(a)


Fig. 4. (a) The HANNYA-SHINGYO. This sutra composed of 262 Chinese characters is an extremely condensed form of a great scripture termed the HANNYA Sutra and thus contains the quintessence of the original text. (b) Mandala pattern generated from (a), where $(S, f, f / N[\%])=(1,416,74.8),(2,84,15.1),(3,24,4.3)$, $(4,8,1.4),(6,4,0.7),(9,8,1.4),(20,4,0.7),(48,4,0.7)$, and $(108,4,0.7)$ with $N=556(m=9)$.
(b)


Fig. 4. (continued).
mandala is composed of 528 elements (i.e., $N=528$ ). For an example of highly dense pattern generation, the RISHU Sutra (see, e.g., MATSUNAGA, 1992) given in Fig. 6(a) is chosen. As seen in Fig. 6(a), this sutra is composed of the most sentences among the three sutras. The mandala pattern generated from Fig. 6(a) is shown in Fig. 6(b), where the mandala is divided into 856 elements $(N=856)$.

The mandala pattern arising from the Chinese poem BAITAN'O (given in Fig. 7(a)) that was composed by Hakukyoi (TABEI, 1990) is exhibited in Fig. 7(b).

The Tale of Genji is a long fiction authored by Murasaki Shikibu (Lady Murasaki) in the middle of the Heian period. Currently this work is regarded as being composed of the 54 volumes as seen in Fig. 8(a). The mandala pattern generated from Fig. 8(a) is shown in Fig. 8(b). However, there once was a different arrangement for the volumes. The one including KUMOGAKURE in Volume 41 (KANEKO, 1952) is given in Fig. 9(a). The pattern obtained from Fig. 9(a) is displayed in Fig. 9(b). In comparison between the two patterns (Figs. 8(b) and 9(b)) one finds that, though their aspects differ each other, there is no substantial difference between them. Aside from the original text, to date the Tale of Genji has reproduced several arranged digests. For an example of them, we herein consider the one written by Kawaguchi (1962). The sequence of the volumes and the mandala

## 《華厳一乗法界図》



## （a）

Fig．5．A mapped seal in the ICHIJO－HOKKAI Figure in the KEGON Scriptures．This map sketches the path from the start，dharma，to the goal，Buddha，at which spiritual enlightenment will be attainable．（a）Original．
（b）Modification for reading．（c）Mandala pattern generated from（b），where $(S, f, f / N[\%])=(1,372,70.4)$ ， $(2,76,14.4),(3,20,3.8),(4,28,5.3),(6,12,2.3),(7,12,2.3),(12,4,0.8)$ ，and $(19,4,0.8)$ with $N=528(m$ $=8$ ）
pattern are seen in Figs．10（a）and（b），respectively．
The literary works composed of a quantity of volumes comparable to the Tale of Genji can also be found in the Chinese classics．Here we notice Han Fei Tzu（Fig．11（a）；Kanaya， 1994），which includes 55 volumes being close to the number of the Tale of Genji．The mandala pattern generated from Fig．11（a）is shown in Fig．11（b）．In general，superposing two sequences allows one to enhance the degree of complexities of the pattern．For instance，a texture using the volume titles of Han Fei Tzu and those of the Tale of Genji is displayed in Fig．12．The two original sequences，$\{\mathrm{H} 1, \mathrm{H} 2, \ldots, \mathrm{H} 55\}$ and $\{\mathrm{G} 1, \mathrm{G} 2, \ldots, \mathrm{G} 54\}$ ， are merged as


Fig. 5. (continued).

$$
\begin{equation*}
\{\mathrm{H} 1, \mathrm{G} 1, \mathrm{H} 2, \mathrm{G} 2, \ldots, \mathrm{G} 53, \mathrm{H} 54, \mathrm{G} 54, \mathrm{H} 55\}, \tag{2}
\end{equation*}
$$

where $\mathrm{H} i(i=1,2, \ldots, 55)$ represents the $i$-th chapter in Han Fei Tzu and Gi $(i=1,2, \ldots, 54)$ represents the $i$-th volume in the Tale of Genji.

The Japanese classic writing SHOBOGENZO (Mizuno, 1990) authored by Master Dogen is composed of 75 chapters, the number of which exceeds substantially those of the


Fig. 5. (continued).
two works above. Their titles are written in Fig. 13(a). The mandala pattern arising from Fig. 13(a) is drawn in Fig. 13(b).

In haiku, which is the shortest Japanese poem of 17 syllables, a seasonable word termed KIGO must be included in a sentence. Figure 14(a) shows the sequence of summery KIGOes in the Buddhist SAIJIKI (IIDA, 1996). The mandala pattern generated from Fig. 14(a) is seen in Fig. 14(b). For the sequence of autumnal KIGOes one can see the results in Figs. 15(a) and (b).

Finally, mandala generation from the names of the 37 Sacred Symbols of the KONGO World (MAtSunaga, 1991) is described. The sequence of the names is given in Fig. 16(a), from which the pattern of Fig. 16(b) has been produced.

## 3. Analysis

### 3.1. Analytical method

To evaluate geometrical features of the mandala pattern, we firstly examine the frequency distribution of areas of the polygonal elements in the pattern. For more detailed characterization the Simpson's diversity index $1-\lambda$ (SIMPSON, 1949) would be useful, where

$$
\begin{equation*}
\lambda=[N(N-1)]^{-1} \sum_{i=1}^{m} f_{i}\left(f_{i}-1\right) . \tag{3}
\end{equation*}
$$

（a）
《大楽金剛不空真実三摩耶経》
〈般若波羅蜜多理趣品〉
1）如是我聞。
2）一時薄伽梵。
じょし加紙
3）成就殊勝一切如来。
いしまわきやふかん
4）金剛加持三摩耶智。

5 ）已得一切如来灌頂宝冠為三界主。

6）已証一切如来。

7）一切智智瑜伽自在。
いぜらちきゃしさん
8）能作一切如来。
かうさくいせんじよらい
9）一切印平等種種事業。
いつ せん以仏とうしようどようしきよう
10）於無尽無余一切衆生界。
よふしんふよいせんじ申うせん加
11）一切意願作業。
いつせいいげん さくきよう
12）皆悉円満。
かんしつてんまん
13）常恒三世。
しようこうさん せ
14）一切時身語意業。
いつせんししんきょいきよう
15）金剛大毘盧遮那如来。きんこうたい ひろしゃたじよらい
16）在於欲界他化自在天王宮中。
さいよよう加にたくわしさんてんのうき䇶うう
17）一切如来常所遊処吉祥称鄚。
い邯じよらいしようそうしよきつ しようしよう たん
18）大摩尼殿。
たいまじてん
19）種種間錯。
しようじよう功れ さく
20）鈴鐸総幡微風摇撃。 れにたくそうばんひふうようね゙き
21）珠䯹瓔珞半満月等而為荘撖。
じまんえいらくはんまんけつとうじいそうげん
22）与八十倶胝菩薩衆俱。よはつじうくちほさつじうく
23）所謂。
そい
24）金剛手菩薩摩訶薩。 きんこうじほさん は 础さ
25）観自在菩薩摩訶薩。 く
26）虚空蔵菩薩摩訶薩。 きよこうそうほさんばき
27）金剛拳萻薩摩訶薩。 きんこうけんほさんばき加さ
28）文殊師利菩薩摩訶薩。 玌じ曲しワほさんばかさ
29）總発心転法輪菩薩摩訶薩。

30）虚空庫菩薩摩訶薩。

32）与如是等大菩薩衆。 よじょしとうたい氏さつじう
33）恭敬囲繞而為説法。 きようけいいじようじいせつはう
34）初中後善文義巧妙。 そち 5 こう せん 紬きこう ひよう

36）説一切法清浄句門。 せ以つせん㫙 世ん せんくもん
37）所謂。
そい
38）妙適清浄句是菩薩位。
ひようてきせい せんくしほさい
39）欲箭清浄句是菩薩位。
よくせんせんせんくしほさい
40）触清浄句是菩薩位。
41）愛縛清浄句是菩薩位。
そく邯邯くしほさい
あんはくせんせんくしほきい
Fig．6．（a）The RISHU Sutra．The text is known as a scripture being commonly used in the Shingon sect．Note that the term RISHU stands for the reason of things and for that leading to a right thing．（b）Mandala pattern generated from（a），where $(S, f, f / N[\%])=(1,800,93.5),(2,36,4.2),(3,8,0.9),(4,8,0.9)$ ，and $(13,4,0.5)$ with $N=856(m=5)$ ．

42）一切自在主清浄句是菩薩位。
43）見清浄句是菩薩位。 サん せんせんてし庄さい
44）適悦清浄句是萻薩位。 をきごせんせんくし桪さい
45）愛清浄句是菩薩位。 あん せん せんくしほさい
46）慢清浄句是萻薩位。 まん せ U せ いくしほさい
47）荘厳清浄句是菩薩位。そう ザんせんせんくし任さい
48）意滋沢清浄句是菩薩位。いしたくせんせんくしほさい
49）光明清浄句是菩薩位。こうからせんせんくしほさい
50）身楽清浄句是菩薩位。 しんらくせんせんくし任さい
51）色清浄句是菩薩位。 しよくせんせんくしほさい
52）声清浄句是萻薩位。 せん せん せんくしほさい
53）香清浄句是萻薩位。 きよう せんせんくし生さい
54）味清浄句是菩薩位。 ひせひせんくしほさい
55）何以故。 加いこ
56）一切法自性清浄故。 い机生うしせいせんせんこ
57）般若波羅蜜多清浄。 ふわんじゃはらひたせんせん
58）金剛手。 きん こう し
59）若有聞此清浄出生句般若理趣。

60）乃至菩提道場。 約し生ていとうちよう
61）一切蓋障。 いつ せんかんしよう

63）説広積習。 せつこうせきじう
64）必不堕於地獄等趣。 すつぶたよちきょくとうじ
65）説作重罪消滅不難。 せつさくちようさんしようべでなん
66）若能受持日日。じゃくのうじちじつじつ
67）読誦作意思惟。 とくしようさくいしい
68）即於現生証。
そくよけんせんしょう
69）一切法平等金剛三摩地。以 せn 时うべとうきんこうさんまち
70）於一切法皆得自在。よいつせい時う加とくしさい
71）受於無量適悦歓喜。 じゅよりようてきえつ〈かんき
72）以十六大菩薩生。 いじ申うそくたいほさつ 如
73）獲得如来執金剛位。 くかきとくじよ5んじううそこうい
74）時薄伽梵。 し ふわききふわん
75）一切如来大乗現証三摩耶。

76）一切義曼茶羅持金剛勝薩垂。
いや納 むんらちきんこうしよう さつ た
77）於三界中調伏無余。 よ さん 加い施うちようふくなよ
78）一切義成就金剛手菩薩摩訶薩。
いつせんきせんじきんこうし里さんはかさ
79）為欲重顕明此義故。 いよくちよう サんべいしいこ
80）嶣怡微笑左手作金剛慢印。
きい ひしようさじさきんこうまんたん
81）右手抽買本初大金剛作勇進勢。

82）説大楽金剛不空三摩耶心。
せつたいらきんこうふこうさんま

Fig．6．（continued）．


Fig. 6. (continued).

Here $f_{i}(i=1,2,3, \ldots, m)$ stands for the frequency of $S_{i}$ being the area data. This index, which was proposed initially in the context of ecology, is based on probability theory. Suppose that two elements were sampled randomly from the $N$ elements with $m$ categories. In making such sampling a variety of combinations are possible. The index represents the probability of finding the two samples in the same category. As the distribution, i.e., the profile of $\{f\}$ versus $\{S\}$, becomes uniform, the probability $\lambda$ decreases, which leads to the increasing diversity $1-\lambda$. Here we note that the index is bounded within

$$
\begin{equation*}
0 \leq 1-\lambda \leq(N-1)^{-1} N\left(1-m^{-1}\right) . \tag{4}
\end{equation*}
$$

In addition to the Simpson's index, as another approach to analyzing the degree of diversity, we use the Shannon's information entropy $H$ (nat) (ShanNon, 1948) defined in the form

$$
\begin{equation*}
H=-\sum_{i=1}^{m} p_{i} \ln p_{i} \tag{5}
\end{equation*}
$$

with $p_{i}=f_{i} / N(i=1,2,3, \ldots, m)$. The information entropy can be normalized in the form
（a）

| 《売炭翁》 | 白居易 |
| :---: | :---: |
| 1）売炭翁 | 約 たh かう |
| 2）伐薪焼炭南山中 |  |
| 3）満面塵灰煙火色 |  |
| 4）両酸蒼蒼十指黒 | わよう 牰そうそうとしてじつ し くろし |
| 5）売炭得銭何所営 |  |
| 6）身上衣装口中食 | しんじようの い しようこう 5 早うの しよく |
| 7）可憐身上衣正単 | あわれあ ペし しん じょう い まさた ひとえなるを |
| 8）心恵炭賤願天寒 | こころにするの妵きをうかへ てんの さあかんことを如う |
| 9）夜来城外一尺雪 |  |
| 10）暁駕炭車輾冰敝 | あかつきに たん しくに乱てひよう てつをひく |
| 11）牛困人慨日已高 |  |
| 12）市南門外泥中歇 |  |
| 13）确扇両騎来是誰 |  |
| 14）黄衣使者白衫児 | こう いのしくゃ出くさんのぜ |
| 15）手把文書口称勅 | で䋁しよをとりくちに おことかりとしようす |
| 16）廻車叱牛牽向北 |  |
| 17）一車炭重千余斤 | いつしやのすかの おもさ せんよ きん |
| 18）宮使駆将惜不得 |  |
| 19）半足紅糸肖一丈綾 | 生ん ひきのこう しよう いち どようの あや |
| 20）繋向牛頭充炭直 |  |



Fig．7．（a）BAITAN＇O by Hakukyoi．The title of this poem represents an old person who sells charcoal．The composer is a famous Chinese poet in the middle of the Tang period（618－907）．（b）Mandala pattern generated from（a），with $(S, f, f / N[\%])=(1,300,56.4),(2,136,25.6),(3,24,4.5),(4,56,10.5),(6,8,1.5)$ ， $(14,4,0.8)$ ，and $(32,4,0.8)$ with $N=532(m=7)$ ．


Fig. 8. (a) The volume titles of the Tale of Genji. (b) Mandala pattern generated from (a), where ( $S, f, f / N[\%]$ ) $=(1,164,55.4),(2,56,18.9),(3,8,2.7),(4,40,13.5),(6,4,1.4),(16,4,1.4),(20,4,1.4),(34,4,1.4),(42$, $4,1.4),(53,4,1.4)$, and $(87,4,1.4)$ with $N=296(m=11)$.

（a）

| 《新源氏物語》 | 川口松太郎 |
| :---: | :---: |
| 1）藤冝の巻 | ふ以准のまき |
| 2）空蝉 | うつせる |
| 3）夕顔 | 号加古 |
| 4）藤冝ふたたび | 糺烟るたたひ |
| 5）僕流末摘花 |  |
| 6）紅葉の賀 | もあじのが |
| 7）ひと夜遊び | ひとよあそひ |
| 8）ふた夜遊び | ふたよあです |
| 9）花の宴 | はなのえん |
| 10）朧月夜 | あ線ろうきよ |
| 11）車争い | くるまあらその |
| 12）葵 | あむい |
| 13）紫 | あらさき |
| 14）野の宮 | ののおゃ |
| 15）伊勢斎宮 | いせさんくら |
| 16）浮き沈む世姿 | うきじすむよすがた |
| 17）みたび藤薬 | あたひるぜつ任 |
| 18）去り行く藤壺 |  |
| 19）大雷雨 | あおらいう |
| 20）花散里 | はなちる さと |
| 21）別れ別れ | 加が加か |
| 22）須磨 | す |
| 23）源氏帰還 | げんじきがん |
| 24）住吉 | するよし |
| 25）初老感 | しょろう がん $^{\text {d }}$ |
| 26）斎宮入内 |  |
| 27）藤冝逝く | 糺准何 |
| 28）柏木 | 加じき |
| 29）罪障 | ざいしょう |
| 30）消え行く露 | きえしてつ |
| 31）雲隠 | くも施くれ |



Fig．10．（a）The titles of chapters in an Arranged Digest of the Tale of Genji，which was written by Matsutaro Kawaguchi（1899－1985），who is a Japanese novelist being known with，e．g．，AIZENKATSURA．（b） Mandala pattern generated from（a），where $(S, f, f / N[\%])=(1,120,76.9),(2,8,5.1),(3,4,2.6),(4,12,7.7)$ ， $(5,4,2.6),(13,4,2.6)$ ，and $(17,4,2.6)$ with $N=156(m=7)$ ．

$$
\begin{equation*}
h=H / \operatorname{lnm}, \tag{6}
\end{equation*}
$$

which is termed the relative entropy in the context of information theory．
One finds that the patterns exhibited in Subsec． 2.2 contain two different joints：one is shaped like＇+ ＇，the other is shaped like＇$T$ ．＇As a measure of balance between the two different joints we calculate the binary entropy（Shannon，1948）

$$
\begin{equation*}
h_{2}=\left(-p_{+} \ln p_{+}-p_{\mathrm{T}} \ln p_{\mathrm{T}}\right) / \ln 2, \tag{7}
\end{equation*}
$$




Fig. 12. A superimposition between the titles of the Tale of Genji (Fig. 8(a)) and those of Han Fei Tzu (Fig. $11(\mathrm{a}))$, where $(S, f, f / N[\%])=(1,372,61.6),(2,168,27.8),(3,4,0.7),(4,36,6.0),(5,8,1.3),(9,4,0.7)$, $(23,8,1.3)$, and $(30,4,0.7)$ with $N=604(m=8)$.
where $0 \leq h_{2} \leq 1$ and $p_{+}\left(p_{\mathrm{T}}\right)$ stands for the relative frequency of the cross (the T junction); for $p_{+}=p_{\mathrm{T}}=1 / 2, h_{2}=1$. As will be found in the subsequent section, it is important to evaluate the joint distribution in characterizing the 'individuality' of a mandala pattern.

Finally, a method is proposed for measuring a geometrical feature of the outermost boundary of a mandala pattern. As an index dependent on the overall area of a pattern, $S_{\mathrm{tot}}$, and the characteristic length, $a$, we derive

$$
\begin{equation*}
\Gamma=K^{-1}\left(S_{\mathrm{tot}}-2 a^{2}\right)+1 \text { with } K=(\pi-2) a^{2}, \tag{8}
\end{equation*}
$$

where $\pi$ is the Ludolph's number, and the meaning of $a$ is indicated on Fig. 3. Note that $\Gamma$ $\rightarrow 1$ as the contour of a mandala approaches that of the square with side length $\sqrt{2} a$ and area $2 a^{2}$, whereas $\Gamma \rightarrow 2$ as the contour approaches that of the circle with radius $a$ and area $\pi a^{2}$. The $\Gamma$-value beyond the confines of $1 \leq \Gamma \leq 2$ is possible: for $\Gamma<1$ the shape of a mandala becomes starlike, while for $\Gamma>2$ it exhibits a feature of the super-circle characterized by the parameter $\Gamma$. Therefore, with this shape index being calculated, one can quantify the topographic aspect of mandalas.
（a）

| 《正法眼蔵》 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 現成公案 | ザ比ようこうあん | 40） | 栢樹子 | はくじゃし |
| 2 | 摩訶般若㳑 | 羅蜜 | 41） | 三界唯心 | さんがい䊽しん |
|  | ま | はんたゃはらすつ | 42） | 説心説性 | せつ しん せつ しょう |
| 3 | 仏性 ふつ | しょう | 43） | 諸法実相 | しょ䦔じっそう |
| 4 | 身心学道 | しんじんがくとう | 44）仏 | 仏道 | 約 とう |
| 5 | 即心是仏 | そくしんぜい | 45）密 | 密語 | かつ ご |
| 6 | 行仏威儀 | きょうらついいき | 46） | 無情説法 | あじようせつ風う |
| 7 | 一顆明珠 | いつ力が加じ | 47） | 仏経 | ぶつきょう |
| 8 | 心不可得 | しんふかとく | 48） | 法性 | 㖪っしょう |
| 9 | 古仏心 | ごが動 | 49） | 陀羅尼 | たち「 |
|  | 大悟 | だいで | 50） | 洗面 | せんあん |
| 11 | 坐禅儀 | ざ せんき | 51） | 面授 | めんじゅ |
| 12 | 坐禅箴 | ざせんしん | 52）仏 | 仏祖 | 号々 |
| 13 | 海印三昧 | 加いいでんまい | 53）柂 | 梅花 | ＊别 |
| 14 | 空華 | くう ザ | 54） | 洗浄 | せん じょう |
|  | 光明 | こう みょう | 55） | 十方 | どっはう |
| 16 | 行持 | きょうじ | 56）見 | 見仏 | けんち？ |
| 17 | 恁麼 | いも | 57）道 | 遍参 | へん ざん |
| 18 | 観音 | 加のの | 58） | 眼晴 | あん せい |
|  | 古鏡 | こきょう | 59）家 | 家常 | か どょう |
| 20 | 有時 | ゆうじ | 60） | 三十七品咅 | 提分法 |
|  | 授記 | じゅ |  | んじゅうしつ |  |
|  | 全機 | せん き | 61） | 龍吟 | り切ぎん |
| 23 | 都機 | つ き | 62） | 祖師西来意 |  |
|  | 画餅 | D UK |  |  | そしせん5いい |
| 25 | 渓声山色 | 㫙せいさんしょく |  | 発菩提心 |  |
| 26 | 仏向上事 | 約こうじょうじ |  | 優㟺華 | うとん＊ |
| 27 | 夢中説夢 | あちゃうせつ あ |  | 如来全身 | たょらいせんしん |
| 28 | 礼拝得髄 | 5いせいとく馬い | 66） | 三昧王三昧 |  |
|  | 山水経 | さん すいきょう |  |  | まい おう ざん あい |
| 30 | 看経 | 加 きん | 67）転 | 転法輪 | てん过う サん |
| 31 | 諸悪莫作 | しょあくまくさ | 68） | 大修行 | たいじきょう |
| 32 | 伝衣 | でん え | 69） | 自証三昧 | じしょう さんん まい |
|  | 道得 | どうて | 70） | 虚空 | こくう |
|  | 仏教 | ふっきょう |  | 鉢孟 | 旺う |
| 35 | 神通 | じん うう | 72） | 安居 | あんで |
| 36 | 阿羅漢 | あらが加 | 73） | 他神通 | たじんうう |
| 37 | 春秋 | しゅんじゅう | 74） | 王索仙陀粱 |  |
|  | 葛藤 | 加」とう |  |  | おう さく せんたせ |
| 39 | 嗣書 | ししょ | 75）出 | 出家 | しやす |

Fig．13．（a）The titles of chapters in Master Dogen＇s SHOBOGENZO．With this writing，Dogen（1200－1253） stated the spirit of the Buddhism．（b）Mandala pattern generated from（a），where（ $S, f, f / N[\%])=(1,412$ ， $79.2),(2,64,12.3),(3,12,2.3),(4,12,2.3),(7,4,0.8),(8,4,0.8),(16,4,0.8),(17,4,0.8)$ ，and $(19,4,0.8)$ with $N=520(m=9)$ ．


Fig. 13. (continued).

### 3.2. Results

First, numerical results are presented for the mandala pattern of the Sexagenary Cycle (Fig. 3). The mode, Mo, and the range, $R$, of the area data $\{S\}$ are derivable as $M o=1$ and $R=34$, respectively. Here $M o$ and $R$ stand for, respectively, the most frequent value and the difference between the maximum and the minimum of the data. The Simpson's diversity index, the Shannon's information entropy, and the relative entropy, respectively, become

$$
\begin{equation*}
1-\lambda=0.463, H=0.958 \text { nat, } h=0.492 \text {. } \tag{9}
\end{equation*}
$$

The binary joints distribute as

$$
\begin{equation*}
\left(p_{+}, p_{\mathrm{T}}\right)=(50.7,49.3) \text { in } \% \text {, } \tag{10}
\end{equation*}
$$

from which we calculate

$$
\begin{equation*}
h_{2}=1.00, \tag{11}
\end{equation*}
$$

indicating that in Fig. 3 the joints are balanced each other.
The shape parameters, $a$ and $S_{\text {tot }}$, are derived as

| 《仏教歳時記／夏の季語》 |  |
| :---: | :---: |
| 閶魔堂大念仏 | 山法師の花 やま断しの好 |
| えんませうたい玑らい | 芥子坊主 けし质ずす |
| 藤切り会式 糺きりえしき | 金剛証寺開山忌 |
| 仏母会 約もえ | こん もう しょう じ力ん ざんき |
| 薪能 たききのう | 愛染祭 あいせんまつワ |
| 練供養 加くよう | 半夏生 はんひしょう |
| 安居 あんご | 栄西忌 えいさんき |
| 夏断 ザ たち |  |
| 夏書 びがき | 吉野の蛙飛 よしのの約どとひ |
| 夏花 ザ比な | 鬼灯市 咕ぢきいち |
| 夏念仏なつ かんふつ | 四万六千日 し まん ろくせんにち |
| 千団子 せん たh 己゙ |  |
| 団扇撒 うちDまき | 座頭の納涼 ざとうの馬み |
|  | 虫干 札 政 |
| ひとつやいと | きゅうり封じきうりふうじ |
| 峰入 が的 | 志度寺祭じどざさ口 |
| 擬宝珠の花ひ洔うしの如 | 閶魔参 劫まますが |
| 夏遍路なつへんろ | 恐山大祭 あるれ さんたいさい |
| 施米 せまい | 天の橋立祭あまの糺たてまつ |
| 伝教会 でん きょうえ | 雨乞 あま ごい |
| 鑑真忌 $\ddagger$ じん じん | 花火線香 旳ひせんこう |
| 源信忌 ビんしんき | 走馬燈 そうまとう |
|  | 干飯 乱いい |
| かっぱ祭りかっはまつワ | 蓮 は |
| 青葉祭 あ古はまつワ | 沙羅の花 しゃらの約 |
| 鞍馬の竹伐 くらまのたけきり | 仏桑花 ふっそうも゙ |
| 鬼太鼓 㕤でこ | 慈悲心鳥 ビひしんちょう |
| 蟻地獄ありじごく | 仏法僧 或つ罭うそう |
| 真菰まこも | 写経会 しゃきょうえ |
| 菩提樹の花ぼせいじゅのば |  |
| 優量華うど |  |

Fig．14．（a）A juxtaposed sequence of the summery KIGOes in the Buddhist SAIJIKI for haiku．（b）Mandala pattern generated from $(\mathrm{a})$ ，where $(S, f, f / N[\%])=(1,312,65.0),(2,120,25.0),(3,20,4.2),(4,8,1.7),(6$ ， $8,1.7),(19,4,0.8),(22,4,0.8)$ ，and $(58,4,0.8)$ with $N=480(m=8)$ ．

$$
\begin{equation*}
a=18, S_{\mathrm{tot}}=732 \tag{12}
\end{equation*}
$$

With $N=276$ and $S_{\text {tot }}=732$ ，we obtain，as the average area of the elements，$S_{\text {tot }} / N=2.65$ ， and，as the number density of the pattern，$N / S_{\text {tot }}=0.377$ ．On substitution of Eq．（12）into Eq．（8）we evaluate

$$
\begin{equation*}
\Gamma=1.23 \tag{13}
\end{equation*}
$$

Note that the above results of the characteristic values could be a criterion for discussing features and complexities of mandalas．


Fig. 14. (continued).

In Subsec. 2.2 mandala patterns generated from the three sutras have been shown in Figs. 4(b), 5(c), and 6(b). For comparison, numerical results for the characteristic indices are listed in Table 1. First, one may notice that the pattern of Fig. 6(b) exhibits exceptionally small indices of diversity, and thus, is sharply discriminated from the other ones. This can also be seen in the joint distribution; the binary entropy $h_{2}$ is pronouncedly smaller than those of the other patterns. The pattern of Fig. 5(c) is noteworthy in its largest value of $\Gamma$, which indicates that among the three patterns the mandala of Fig. 5(c) is most close to that of the circle with radius $a$. Lastly, a feature of Fig. 4(b) should be mentioned. In this pattern the largest range of the area data is seen $(R=107)$. Indeed the magnitude of $R$ is found to be an order of magnitude larger than the other two. This property of Fig. 4(b) is consistent with the fact that it possesses the largest $S_{\text {tot }}$ among the three.

Subsequently, numerical results for the pattern generated from the Chinese poem BAITAN'O (Fig. 7(b)) are presented. Here, $(M o, R)=(1,31)$ for $\{S\}$; with $N=532$ and $S_{\text {tot }}$ $=1100$, one calculates $\left(S_{\text {tot }} / N, N / S_{\text {tot }}\right)=(2.07,0.484)$. The diversity indices are $1-\lambda=0.604$, $H=1.19$ nat, and $h=0.609$; the joints distribute as $\left(p_{+}, p_{\mathrm{T}}\right)=(66.7,33.3) \%$, which leads
（a）

| 《仏教歳時記ノ秋の季語》 |  |
| :---: | :---: |
| 虫送 狄 かくり | 魂送 たまわくり |
| 解除会 びじょえ | 百八燈 ひゃく出ったい |
| 幽霊祭切れいまつワ | 送火 かくりひ |
| 夜念仏 よ あん おつ | 大文字 Eん もんじ |
| 六道参 ろく とうまいつ | 衝突入つといか |
| 不断経 ふ だん きょう | 盆過 灁すき |
| 六斎念仏ろくさん加ろう | 万燈会 まんどうえ |
| 千日参 せんたちまいす | 戸津說法 とうせっぱう |
| 生身魂いきあたま | 千燈供飱 せんとうくよう |
| 刺鲭 さし さば | 地蔵盆 じそう断 |
| 盆礼 任ん \＃い | 大覚寺大日会 |
| 盆路 断わち | 約加くじ約にちえ |
| 墓参 は加まいつ |  |
| 掃苔 そうたい | 法師蝉 ほう |
| 西瓜提灯 朝加ちょう ちん | 鉦吅 加たたたき |
| 盆燈籠 任ん どうろう | 八朔盆 世つさく䢻 |
| 盆用意 断ようい | 放生会 比うどうえ |
| 草の市くさのいち | 風の盆 加せの明 |
| 茄子の馬 す のうま | 龍口法難会 たつのくち罒うなんえ |
| 苧殻 あから | 薄念仏会すすきるん ふつ |
| 門火 加光 ひ | 観月讚仏会 加れけつさんふつえ |
|  | 秋遍路 あきへんろ |
| 孟蘭盆うら臥 | 曼珠沙華 まん じゅしゃけ |
| 魂祭 たままつり | 秋彼岸 あき すがん |
| 盆の寺 断のてら | 遊行忌きょうき |
| 摂待 せつたい | 道元忌 とう げん き |
| 棚経僧 たなきょうそう | 人形供養 たん きょうくよう |
| 盆東風 断ごち | 菩提子ぼたいし |
| 盆波 任んなね | 数珠玉 じ或だき |
| 盆の月 比しのつき | 光明真言会 こう あようしん ざん え |
| 盆花 㖪ん断 | 菊の節句きくのせっく |
| 盆螕 妇ん が | 太秦の牛祭 うずまさのう $\begin{aligned} & \text {（ }\end{aligned}$ |
| 盆踊 断わせぜ） | 御命講 あありこう |
| 盆芝居 断しまい | 題目立 たいもくたて |
| ほうか | 鬼子母神祭 きしもじょまつワ |
| 鬼来迎きらいごう | 菊供䑆 きくくよう |
| 盈休 臨阬み | 釈迦念仏会しゃ加加ぶつえ |
| 解夏 ザげ | 観音草 加んのんそう |
| 施餓鬼せがき | 理供養 ためきくよう |
|  |  |
| 燈籠流 とうろうなが | 正倉院曝涼 |
|  | しょうそう Wん はくりょう |

Fig．15．（a）A juxtaposed sequence of the autumnal KIGOes in the Buddhist SAIJIKI for haiku．（b）Mandala pattern generated from（a），where $(S, f, f / N[\%])=(1,544,81.0),(2,72,10.7),(3,16,2.4),(4,28,4.2),(6$ ， $4,0.6),(8,4,0.6)$ ，and $(14,4,0.6)$ with $N=672(m=7)$ ．


Fig. 15. (continued).
to $h_{2}=0.918$. With $a=22$ and $S_{\text {tot }}=1100, \Gamma=1.24$.
The results of the patterns generated from the original (Fig. 8(b)) and its variants (Figs. $9(\mathrm{~b})$ and $10(\mathrm{~b})$ ) of the Tale of Genji are listed in Table 2. Through comparison among the three the pattern of Fig. 10(b) shows a sharp contrast to the other two. For instance, because of the shortest sequence (see Fig. 10(a)) the total area of Fig. 10(b) is much smaller than those of Figs. 8(b) and 9(b). This property of Fig. 10(b) is responsible for reducing the indices of diversity. There are features in common between Figs. 8(b) and 9(b). Among them the distribution of the two joints is worth noting; one finds that $p_{+}<p_{\mathrm{T}}$, indicating that in Figs. 8(b) and 9(b) the tee junctions dominate the crosses.

The results of the patterns shown in Figs. 11(b) and 12 are summarized in Table 3, where the effect of the superimposition between the Tale of Genji (Fig. 8(a)) and Han Fei Tzu (Fig. 11(a)) is analyzed. It is found that with merging the two sequences, in addition to $N, a$, and $S_{\text {tot }}$, all the diversity indices including $h_{2}$ as well as the magnitude of $\Gamma$ can be enhanced. This finding is consistent with the fact that Fig. 12 appears highly dynamical and is much more complicate than Fig. 11(b).

Next, numerical results for the pattern generated from the Master Dogen's SHOBOGENZO (Fig. 13(a)) are given. The diversity indices are $1-\lambda=0.356, H=0.803$ nat, and $h=0.366$. The binary joints distribute as $\left(p_{+}, p_{\mathrm{T}}\right)=(60.9,39.1) \%$, which yields $h_{2}$ $=0.965$. In Fig. 13(b), $a=21$ and $S_{\text {tot }}=892$, which lead to $\Gamma=1.02$. Here we note that the number of elements, $N=520$, in Fig. 13(b) is comparable to those in Figs. 5(c) and 7(b) ( $N$

| 《金剛界三十七尊名》 |  |
| :---: | :---: |
| 1）金剛鈴菩薩 | ごんごうかい违さつ |
| 2）金剛鎖菩薩 | こんごうさほさう |
| 3）金剛索菩薩 | こんじうさく回さつ |
| 4）金剛鉤菩薩 | こんごうこうは さつ |
| 5）金剛塗菩薩 | こんごすぎ住さ |
| 6）金剛灯菩薩 | こんごうとうほ さつ |
| 7）金剛花菩薩 | こんごうかもざ |
| 8）金剛香菩薩 | こんごうこうほざ |
| 9）金剛舞菩薩 | こんごうる比さつ |
| 10）金剛歌菩薩 | こん ごう加生さつ |
| 11）金剛鳌菩薩 | こん ごうまんほさつ |
| 12）金剛嬉菩薩 | こんじうきはさつ |
| 13）金剛拳菩薩 | こんごうげんほさつ |
| 14）金剛牙菩薩 | こんごうげほさ |
| 15）金剛護菩薩 | こんだうご回さつ |
| 16）金剛業菩薩 | こんごうごうほさつ |
| 17）金剛語菩薩 | こんごうごは任さつ |
| 18）金剛因菩薩 | ごんごういんほざ |
| 19）金剛利菩薩 |  |
| 20）金剛法菩薩 | こんごうほうぼき |
| 21）金剛笑菩薩 | こん ごう しょう（里さつ |
| 22）金剛幢菩薩 | こんごう とう（ さ |
| 23）金剛光菩薩 | こん ごうこうぼき |
| 24）金剛宝菩薩 | こんごう㖘扫さつ |
| 25）金剛喜菩薩 | こん ごう き妊さつ |
| 26）金剛愛菩薩 | こんごう あい活さつ |
| 27）金剛王菩薩 | こんごう おう 居ざ |
| 28）金剛薩垂菩薩 | こんごう さった時さつ |
| 29）羯磨波羅蜜菩薩 | 加つまはらすつほさつ |
| 30）法波羅蜜菩薩 | 断はらみつ国さつ |
| 31）宝波羅蜜菩薩 | 娃はらかつぼさつ |
| 32）金剛波羅蜜咅薩 | こんごうはらすつ建さつ |
| 33）不空成就如来 | ふくらじょうどゅによらい |
| 34）阿弥陀如来 | あなたよらい |
| 35）宝生如来 | 住うしょうたよ5い |
| 36）阿門如来 | あしゃくによらい |
| 37）大日如来 | たいたちた」らい |

Fig．16．（a）The names of the 37 Sacred Symbols of the KONGO World in the Esoteric Buddhism．（b）Mandala pattern generated from（a），where $(S, f, f / N[\%])=(1,316,58.5),(2,160,29.6),(3,8,1.5),(4,32,5.9),(5$ ， $12,2.2),(11,8,1.5)$ ，and $(17,4,0.7)$ with $N=540(m=7)$ ．
$=528$ and 532，respectively）．Comparison among the three patterns will be made in the subsequent section．

The results of the patterns shown in Figs．14（b）and 15（b）are compared in Table 4．It can be seen that the two patterns exhibit a marked contrast both in the indices of diversity and in the shape parameter．In particular，the exceptionally small value of $\Gamma$ for Fig．14（b）


Fig. 16. (continued).
( $\Gamma=0.773$ ) would be of interest. As mentioned in the explanation of Eq. (8), the $\Gamma$-value less than unity represents a starlike mandala as seen in Fig. 14(b). Also note that in Fig. 15(b) all the diversity indices including $h_{2}$ become smaller than those calculated for Fig. 14(b).

Lastly, analyzed results for Fig. 16(b) are described: $N=540, m=7 ; M o=1, R=16$; $H=1.09$ nat, $h=0.559,1-\lambda=0.566 ; h_{2}=0.988 ; a=22, S_{\text {tot }}=1004$, and $\Gamma=1.07$. Here one notices the value of $N$ being comparable to those seen in Figs. 5(c), 7(b), and 13(a). Discussion will be made in the subsequent section.

## 4. Discussion

To discuss the quality of the mandala patterns, firstly comparison will be made between the pattern shown in Fig. $3(N=276)$ and that shown in Fig. 11(b) $(N=272)$; as written in the bracket the numbers of elements composing the mandalas are in good agreement. Visually, one may have an impression that Fig. 3 is dynamic, whereas Fig. 11(b) is static. This can be explained through comparative analysis among the key parameters, specifically, the number density of polygonal elements, the relative entropy (or the Simpson's index of diversity), and the $\Gamma$-value, which are reproduced by

Table 1. Comparison among characteristic indices of mandalas generated from sutras.

|  | HANNYA <br> (Fig. 4(b)) | HOKKAI <br> (Fig. 5(c)) | RISHU <br> (Fig. 6(b)) |
| :--- | :---: | :---: | :---: |
| $m$ | 9 | 8 | 5 |
| $N$ | 556 | 528 | 856 |
| $S_{\text {tot }}$ | 1488 | 976 | 980 |
| $S_{\text {to }} / N$ | 2.68 | 1.85 | 1.14 |
| $N / S_{\text {tot }}$ | 0.374 | 0.541 | 0.873 |
| $M o$ of $S$ | 1 |  |  |
| $R$ of $S$ | 107 | 1 | 18 |
|  |  |  | 12 |
| $1-\lambda$ | 0.416 | 0.478 | 0.125 |
| $H$ (nat) | 0.902 | 1.05 | 0.309 |
| $h$ | 0.411 | 0.506 | 0.192 |
|  |  |  |  |
| + joint | $57.7 \%$ | $61.4 \%$ | $80.0 \%$ |
| T joint | $42.3 \%$ | $38.6 \%$ | $20.0 \%$ |
| $h_{2}$ | 0.983 | 0.962 | 0.722 |
|  |  |  |  |
| $a$ | 26 | 20 | 21 |
| $\Gamma$ | 1.18 | 1.39 | 1.19 |

Table 2. Comparison among characteristic indices of mandalas generated from three versions for the volume titles of the Tale of Genji.

|  | Modern <br> (Fig. 8(b)) | Old <br> (Fig. 9(b)) | Digest <br> (Fig. 10(b)) |
| :--- | :---: | :---: | :---: |
| $m$ | 11 | 11 | 7 |
| $N$ | 296 | 304 | 156 |
| $S_{\text {tot }}$ | 1492 | 1468 | 336 |
| $S_{\text {to }} / N$ | 5.04 | 4.83 | 2.15 |
| $N / S_{\text {tot }}$ | 0.198 | 0.207 | 0.464 |
| $M o$ of $S$ | 1 |  |  |
| $R$ of $S$ | 86 | 84 | 1 |
| $1-\lambda$ |  |  | 16 |
| $H$ (nat) | 1.42 | 1.639 | 0.745 |
| $h$ | 0.591 | 0.701 | 0.400 |
|  |  |  | 0.927 |
| + joint | $43.4 \%$ | $45.8 \%$ | $54.2 \%$ |
| T joint | $56.6 \%$ | $54.2 \%$ | $45.8 \%$ |
| $h_{2}$ | 0.987 | 0.995 | 0.995 |
|  |  |  |  |
| $a$ | 27 | 26 | 13 |
| $\Gamma$ | 1.04 | 1.15 | 0.990 |

Table 3. Comparison between characteristic indices of mandalas generated from the titles of chapters in Han Fei Tzu (Fig. 11(b)) and those of the superimposition with the volume titles of the Tale of Genji (Fig. 12).

|  | Han Fei Tzu <br> (Fig. 11(b)) | Superimposition <br> (Fig. 12) |
| :--- | :---: | :---: |
| $m$ | 8 | 8 |
| $N$ | 272 | 604 |
| $S_{\text {tot }}$ | 980 | 1244 |
| $S_{\text {tol }} / N$ | 3.60 | 2.06 |
| $N / S_{\text {tot }}$ | 0.278 | 0.486 |
| $M o$ of $S$ | 1 |  |
| $R$ of $S$ | 72 | 1 |
|  |  | 29 |
| $1-\lambda$ | 0.531 | 0.540 |
| $H$ (nat) | 1.16 | 1.04 |
| $h$ | 0.559 | 0.499 |
| + joint | $61.2 \%$ | $56.8 \%$ |
| T joint | $38.8 \%$ | $43.2 \%$ |
| $h_{2}$ | 0.963 | 0.987 |
| $a$ | 22 | 23 |
| $\Gamma$ | 1.02 | 1.31 |

Table 4. Comparison between characteristic indices of mandalas generated from the string of the summery KIGOes (Fig. 14(b)) and that of the autumnal KIGOes (Fig. 15(b)).

|  | Summery <br> (Fig. 14(b)) | Autumnal <br> (Fig. 15(b)) |
| :--- | :---: | :---: |
| $m$ | 8 | 7 |
| $N$ | 480 | 672 |
| $S_{\text {tot }}$ | 1088 | 960 |
| $S_{\text {to }} / N$ | 2.27 | 1.43 |
| $N / S_{\text {tot }}$ | 0.441 | 0.700 |
| $M o$ of $S$ | 1 |  |
| $R$ of $S$ | 57 | 1 |
|  |  | 13 |
| $1-\lambda$ | 0.514 | 0.331 |
| $H$ (nat) | 1.02 | 0.723 |
| $h$ | 0.488 | 0.372 |
| + joint | $55.6 \%$ | $71.8 \%$ |
| T joint | $44.4 \%$ | $28.2 \%$ |
| $h_{2}$ | 0.991 | 0.858 |
| $a$ |  |  |
| $\Gamma$ | 25 | 21 |

$$
\left(N / S_{\text {tot }}, h, \Gamma\right)=\left\{\begin{array}{lll}
(0.377, & 0.492, & 1.23) \text { for Fig. 3 }  \tag{14a}\\
(0.278, & 0.559, & 1.02) \text { for Fig. } 11(\mathrm{~b})
\end{array}\right.
$$

As is found from Eq. (14), the largest difference can be seen in the value of $\Gamma$. The larger $\Gamma$-value of Fig. 3 reflects the intricate topography of the contour, while the smaller value of Fig. 11(b) arises from the smooth contour. It should be emphasized again that in the limit of $\Gamma \rightarrow 1$ the outline of mandalas approaches that of the square with side length $\sqrt{2} a$. In addition to the $\Gamma$-value, in order to enhance complexities of mandalas, the number density, $N / S_{\text {tot }}$, should be higher without significantly reducing the diversity indices. It can be concluded that in spite of the relatively small $N$ the pattern shown in Fig. 3 meets the abovementioned requirements.

Subsequently, we take notice of Figs. 8(b) and 9(b); from Table 2 the key parameters are selected:

$$
\left(N / S_{\text {tot }}, h, \Gamma\right)= \begin{cases}(0.198, & 0.591,  \tag{15a}\\ (0.207, & 0.701, \\ (0.15) \text { for Fig. } 8(\mathrm{~b}) \\ \text { for Fig. } 9(\mathrm{~b})\end{cases}
$$

It can be seen from Eq. (15) that there seems to be no significant difference between the number densities; difference can be found in $h$ as well as the $\Gamma$ values. In comparison between the two patterns one would judge that the latter (Fig. 9(b)) is more dynamical than the former (Fig. 8(b)). It is concluded that this observation arises from the difference of $h$ and of the $\Gamma$-value.

In order to confirm the validity of the above discussion, we select five patterns with the same order of magnitude for $N(520 \leq N \leq 556)$ : specifically, Figs. 4(b), 5(c), 7(b), 13(b), and $16(\mathrm{~b})$ are highlighted. To aid comparison the three parameters are specified:

$$
\left(N / S_{\mathrm{tot}}, h, \Gamma\right)= \begin{cases}(0.374, & 0.411,1.18) \text { for Fig. 4(b) }  \tag{16a}\\ (0.541, & 0.506,1.39) \text { for Fig. } 5(\mathrm{c}) \\ (0.484, & 0.609, \\ (0.583, & 0.366, \\ (0.538, & 0.559, \\ (0.02) \text { for Fig. } 7(\mathrm{~b}) \\ \text { for }) \\ \text { for Fig. 13(b) }) \\ (\mathrm{b})\end{cases}
$$

Visually one would have an impression that the patterns shown in Figs. 5(c) and 7(b) are dynamical and/or complicate whereas that shown in Fig. 13(b) is statical; the remainder (Figs. 4(b) and $16(\mathrm{~b})$ ) is intermediate. Here, it appears to the author that the pattern of Fig. $5(c)$ is of particular intricacy, being consistent with the fact that the $\Gamma$-value becomes maximum. In addition, it should be mentioned that, although the $\Gamma$-value is close to unity, the pattern shown in Fig. 16(b) could be regarded as much more intricate than that shown in Fig. 13(b). This result is explainable with the higher entropy for Fig. 16(b). We thus
conclude that to make mandala patterns complicate, the magnitude of entropy maintains a certain level. Indeed, it is found from Eqs.(16) that the entropy becomes maximum for Fig. 7(b). The highest entropy observed for Fig. 7(b) will be responsible for the highly dynamic nature mentioned above. To conclude, in performing detailed diagnosis of the mandalas, the use of the diversity indices such as entropy is necessary in conjunction with the number density and the $\Gamma$-value.

Finally, an analogy of the pattern formation of the mandalas will be mentioned to the formation of a human face. As is well known, all the information about the size as well as the configuration of facial parts such as eyes, eyebrows, ears, a nose, and lips is written on a genetic sequence. This indicates that one-dimensional 'letter' sequences produce a variety of the two-dimensional patterns; their aspects are determined uniquely by the type of sequence being inherent in a person. The mandala generation proposed in this paper might be regarded as nothing but a reproduction of the biological pattern formation.

## 5. Conclusions

The two-dimensional patterns with four-fold mirror symmetry have been generated from texts written with Chinese characters. The drawing is based on the spiral mapping technique, with which one can visualize a statistical property of a text in a two-dimensional region through spirally folding a string of the characters. According to the Jung's classification these symmetrical patterns can be categorized into Type-I mandalas. After illustrating the outline of the method, typical results of the pattern generation have been shown and subsequently analyzed by means of the diversity indices. Specifically, mandala patterns have been generated from a variety of texts such as the HANNYA SHINGYO, the ICHIJO-HOKKAI Figure in the KEGON Scriptures, the RISHU Sutra, a Chinese poem, and strings of words, such as titles of chapters in a literary work. Words have been cited from the Tale of Genji, Han Fei Tzu (KAMPISHI), Master Dogen's SHOBOGENZO, KIGOes in haiku, and eventually the 37 Sacred Symbols in the KONGO World. Comparison has been made among features of the mandalas. Finally, it should be mentioned that the process of the mandala generation presented in this paper might be useful for performing art therapy using mandala (Kellogg et al., 1977; TsuKasaki, 1991; TsuKasaki et al., 1992).

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