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Introduction: In Search of the Golden Mean

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Fine artists, composers, architects, scientists, and engineers have often created their best works by keeping an open dialogue with the natural world. In nature there is a wonderful duality between order and chaos. Careful study of cloud formations or a running stream shows that what at first appear to be random fluctuations in observed patterns are actually subtle forms of order. Mathematics is the best tool that humans have created to study the order of things.

Despite the infinite diversity of nature, mathematics and science have attempted to reduce this complexity to a few general principles. One enigmatic number, the golden mean or golden section, appears and reappears throughout works of art and science. Bela Bartok created much of his music with the use of the golden mean and Fibonacci sequences. Le Corbusier is well known to have used the Modulor, based on the golden mean, to proportion his buildings. References to the golden mean can be found in the notebooks of the artist, Paul Klee. Paul Seurat and Piet Mondrian are also known to have used the golden section in their work. From the scientific side, articles are continuously published in all areas of science and mathematics pertaining to the golden mean. In fact a journal, the Fibonacci Quarterly, has been published on this subject for the past fifty years.

However, MARKOWSKY (1992) has left us with a cautionary tale. Connections to the golden mean have more often than not been alleged without adequate justification, based on superficial measurements. If one is willing to accept an error of as little as 1%, almost anything can be made to fit a preconceived measurement. As a result, it is conventional wisdom that the golden mean proportions the exterior of the Parthenon. Recent work of BULKENS (1999) and KAPPRAFF and MCCLAIN (2005) makes this appear unlikely. I myself created a template from which I measured certain points of tension and focus in many classic paintings and invariably found them to conform to the golden section. On one such occasion, while measuring clear division points in the super-realistic urban scenes of the artist Richard Estes, I found them to be almost exact golden mean proportions. I communicated with the artist and received a polite reply that he was vaguely aware of the golden mean from his college days, but by no means consciously incorporated them into his work. Rather than being disappointed with his response, something more fundamental dawned on me. The golden section presents us with a point that is slightly displaced from the true center by such an amount as to result in the natural state of tension needed to create a work of art and which our brains recognize as pleasing. It is likely that artists have unconsciously built these points of tension into their art. In fact Robert Lawlor has stated:

> An asymmetrical division is needed in order to create the dynamics necessary for progression and extension from unity. The golden mean is the perfect division of unity.

In this issue Janusz Kapusta presents geometrical "snapshots" of the deepest properties of the golden mean in his picture essay, "The Square, the Circle, and the Golden Proportion:

A New Class of Geometrical Constructions." He shows how subtle properties of the golden mean that express themselves in number also have analogues in geometry. He has captured geometric images of the entire family of silver means by using geometry based on the circle and the square. His surprising images are a testament to the singularity of this extraordinary number.

Louis Kauffman addresses the self-referential nature of the golden mean. He shows that its unusual geometry is the result of patterns of self-interaction of infinite re-entering forms of a single symbol know as "the mark" ¬. Here the mark is conceived as an "elementary particle" that can interact with itself to either produce itself or to annihilate itself. The Fibonacci property of the self-interactions of the mark is a link between *Laws of Form* by the communications engineer and philosopher, G. Spencer-Brown, as a way of studying self-referential logic. There are links between Laws of Form, topology, and quantum information theory suggested by this paper.

The only instance in nature that I know of in which the golden mean and its close relative, the Fibonacci sequence has explicitly revealed itself is in the growth of plants or plant phyllotaxis as it is called. As I show in "Growth of Plants: A Study in Number", the golden mean insures that plants grow in such a way as give each floret the most space. Subtle properties of number are used to accomplish this task. In fact Le Corbusier has made the following statement with regard to the golden mean:

Behind the wall, the gods play; they play with numbers of which the universe is made up.

This article suggests that the growth of plants has strong connections to the general theory of dynamical systems and chaos theory. In fact the golden mean imprints itself through a sequence of 1's and 0's known as the rabbit sequence which also makes its appearance in other dynamical systems at the brink of chaos.

In our article "Generalized Binet's Formula, Lucas Polynomials, and Cyclic Constants", Gary Adamson and I present, as actors on a mathematical stage, two families of numbers called silver means the first of which is the golden mean, and families of Pell and Pell-Lucas sequences the first two of which are the Fibonacci and Lucas sequences. These sequences are generated by Fibonacci and Lucas polynomials and lead to periodic sequences of "cyclic constants" that have connections to the energy levels of hydrocarbons on the one hand and questions of the constructability of regular polygons on the other.

In another article, "Golden Fields, Generalized Fibonacci Sequences and Chaotic Matrices", my co-workers and I show that the diagonals of each regular polygon with an odd number of sides forms an algebraic system that we call "golden fields" and leads to another generalization of Fibonacci sequences and to connections between the wonderful order of regular polygons and chaos theory as described by the Mandelbrot set.

Haruo Hosoya has discovered sequences of graphs related to the golden mean and Fibonacci and Lucas sequences that describe the structure and energy levels of hydrocarbon chemistry. He assigns a topological invariant called its Z-value to each graph, where the ratio of consecutive Z-values approach the golden mean. Two of his articles: "Some Graph-theoretical Aspects of the Golden Ratio: Topological Index, isomatching Graphs, and Golden Family Graphs", and "Sequences of Polyomino and Polyhex Graphs whose Perfect Matching numbers are Fibonacci or Lucas Numbers" are presented in this special issue.

The starting point of his analysis is one of the most fundamental structures of mathematics, Pascal's triangle. Mathematical recreations with polyominoes and polyhexes play an important role in his work.

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