Research of Pattern Formation, edited by R. Takaki, pp. 1–10.

Introduction to Science of Pattern Formation

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Abstract. A brief historical survey is given on research of pattern formation as a fundamental science. Recent needs and developments in Japan of this field are explained including the establishment of The Society for Science on Form, Japan. It is suggested that a general framework of the science of pattern formation is composed of the three kinds of topics, the geometrical aspects of forms, the mechanisms of pattern formation and measurements of forms (morphometry). As a branch of the morphometry, the role of the stereology is stressed.

1. Historical Survey

Let us begin this article with recollecting works in modern ages which are important in view of the science of pattern formation. The term "science of pattern formation" is yet neither popular nor properly defined. The concept of this science owned by the present author will be made clearer in the following by introducing activities of scientists in this field.

One of the most attractive forms in nature is the crystal. Crystal forms began to be investigated since the law of constant interfacial angle was found in 1669 by N. Stensen. This law assirts that angles between faces are same for the crystals with the same components under the same pressure and the same temperature. Although microscopic crystal structures became the main interest of the crystallography in this century, problems of macroscopic forms remained attractive owing to their beautiful appearances and also relations to material technology.

A. L. Wegener, German geophysicist, proposed in 1912 the theory of continental drift, which has been established and developed to the plate techtonics. It should be noted that F. Bacon also paid attension to the coastal shapes of the continents. The theory of continental drift is an important landmark of the history of science because it challenged the fixed idea of stationary continent. It is also a high-light in the science of pattern formation in a sence that the study of coastal patterns lead to a great discovery.

In the field of chemistry several interesting phenomena have been studied though not as main topics, such as the Lichtenberg's figure by G. C. Lichtenberg in 1777 (an electric discharge with dendritic trace in a photosensitive plate), the Riesegang ring by R. E. Riesegang in 1896 (concentric rings of presipitations in the solution of potassium dichromate and silver nitrate), and the BZ reaction by B. P. Belousov in 1959 and A. M. Zhabotinsky in 1967 (an oxidation-reduction reaction of maronic acid in a solution containing several other components). These phenomena tell that the chemistry is closely related to pattern formation. Figure 1 shows a pattern of the BZ reaction taken by Miike (1988), where the reaction proceeded with oscillation and the spiral pattern moved to radial direction (see Fig. 1).

In biology basic study of forms was initiated by J. W. Goethe, who first used the term "morphology". But, most works in morphology were qualitative description of shapes. It was D. W. Thompson, English scientist of natural history, who cast mathematical and physical spotlights on biological patterns. His monograph "On Growth and Form" (1969) has played a role of stimulating many people to creative

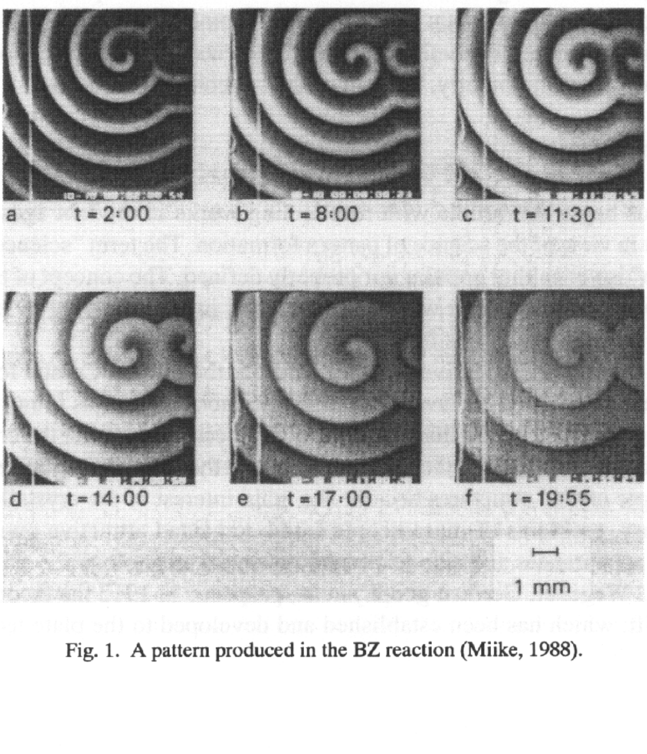

Fig. 1. A pattern produced in the BZ reaction (Miike, 1988).

works (Fig. 2.). The recent computer analysis of early stages of development of the newt, an animal belonging to amphibia, by Gordon and Jacobson (1978) is a good example of works succeeding Thompson's method.

In hydrodynamics, which is a branch of classical physics, we have a lot of works which are considered to be motivated by an interest in pattern formation. The thermal convection in a horizontal layer heated from below is known to form a periodic cell structure, which is named Bénard convection after H. Bénard who observed this phenomenon. The periodic shedding of regular vortices into the downstream region of a circular cylinder placed in a uniform flow is one of the beautiful patterns produced by the fluid inertia effect (see Fig. 3). These vortices were observed by Bénard in 1906 and studied theoretically in 1911 by T. Karman, hence they are called Karman vortex. The Taylor vortex observed and analysed by G. I. Taylor in 1923 is also an example of topics which attracted people because of its pattern, although its practical importance is rather limited.

The most important nature of those patterns is that they are produced in systems out of thermal or mechanical equilibrium. Theoretical treatment of pattern formation in the thermal inequilibrium is made by I. Prigogine and his group (Gransdorff and Prigogine, 1971). The term "dissipative structure", which is given for patterns produced through dissipative effects, has now become popular. In these phenomena an energy is input continuously, which sustains the orderly structures. The crystal growth, the chemical patterns, the Bénard convection, etc. are dissipative

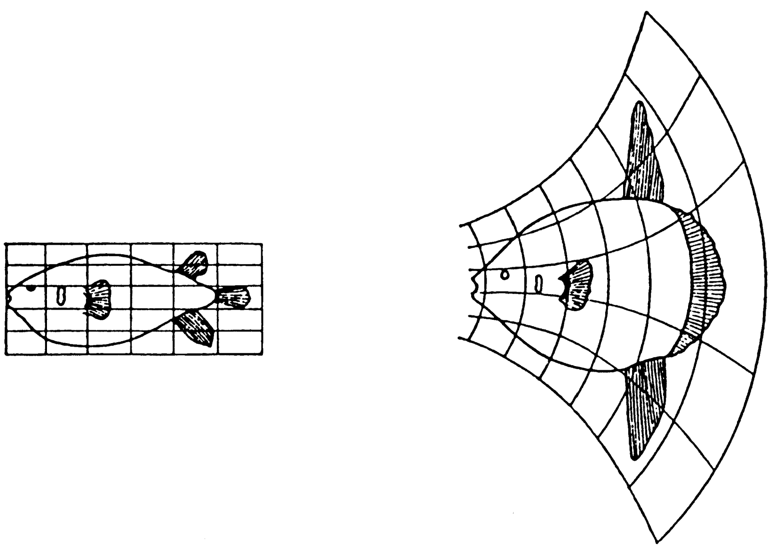

Fig. 2. Transformation of fish shapes after Thompson's coordinate method (1969). Left: porcupine-fish, right: sun-fish.

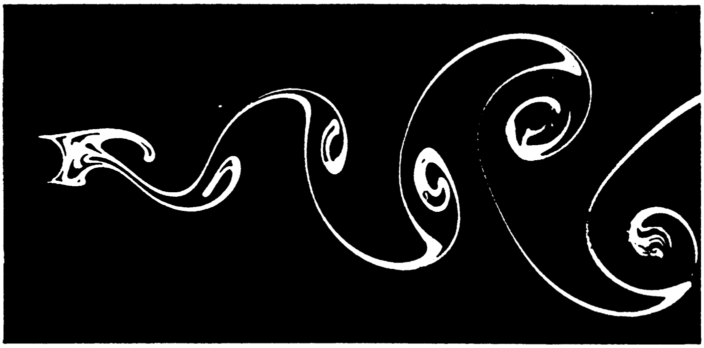

Fig. 3. Karman vortex street in the flow past a circular cinlinder (Taneda, 1988).

structures. On the other hand, H. Haken, German physicist, gave a theoretical basis for laser oscillation in terms of nonlinear mechanics, as described in his monograph (1981). Investigation of definite orders in systems with nonlinear effect and continuous energy input constitutes an important branch of the science of pattern formation.

As for contributions from mathematics, the following two findings are worth mentioning. When R. Thom and his coworkers began to discuss on formations of biological patterns in terms of "catastrophy", a shock propagated among scientists in the world (Thom, 1975). Actually, the catastrophy theory is a classification of solutions of certain types of differential equation. However, it depends on future development whether this theory plays an important role as a model of pattern formation phenomena. The second contribution was given by Mandelbrot (1977). The concept "fractal" formulated by him has aquired a position as one of important parameters characterizing patterns in nature. The reason why fractal is loved so much may be that it is clearly defined, and that it can be applied even to random patterns. Moreover, a trend is seen recently in biological sciences to calculate fractal dimensions for analyses of structures of blood vessels or other human organs.

We now turn to the development of science of pattern formation in Japan. The name which should be given first is Torahiko Terada, a physicist at the beginning of the 20-th century. Besides his original field, X-ray diffraction, he investigated various phenomena which are seen in usual life, such as the electric discharge, the fractures in glass, the form of "kompeito" (a sugar candy with a lot of vertices on its surface), etc. His activity was not always estimated favorably because it was not looked upon as an orthodox science. But, his attitude of research was nevertheless followed by several scientists, as introduced below.

Ukichiro Nakaya studied the formation of snow crystal, the frozen soil and the icing, i.e., a growth of ice on an object. His Nakaya-diagram to classify shapes of snow flakes is a pioneering work in this field, which is revised now and replaced by the Kobayashi-diagram (Kobayashi, 1961). Morizo Hirata (1975) was mainly engaged in phenomena of fractures in glass, plastic, or several other materials by the use of statistical methods (Fig. 4). The study of kompeito is succeeded by Morikazu Toda (1981), though the mechanism of vertex formation is still unknown (Fig. 5).

As is seen from these examples, recent development of the science of pattern formation in Japan can be looked upon as a recognition of the "Terada's physics".

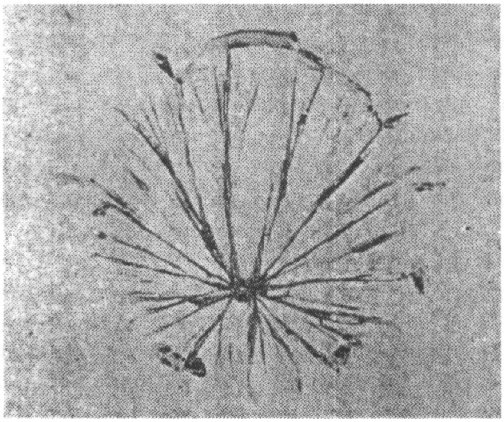

Fig. 4. Crackes in a plateglass heated at the center (Hirata, 1975).

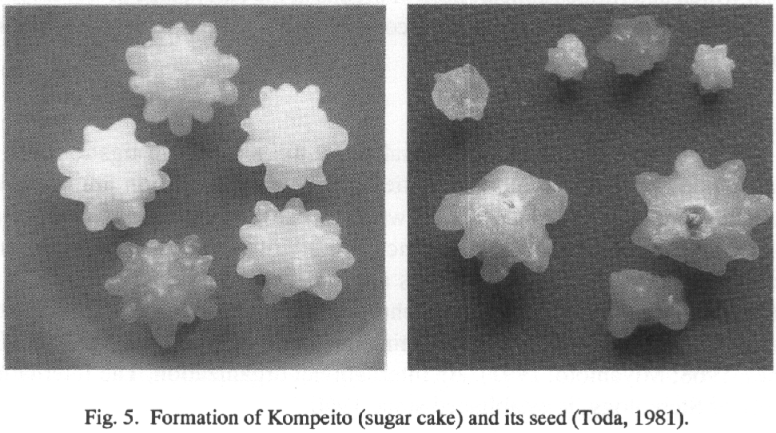

Fig. 5. Formation of Kompeito (sugar cake) and its seed (Toda, 1981).

2. Recent Situation

Problems concerned with patterns are now investigated much in various fields. There are two reasons for its increasing interest. One is the recent development of high-speed computer, which enables processing of digitized images. Another is the ever growing need to study patterns. Some examples of this second factor is given below.

2.1 New materials

A lot of new materials, such as amorphus, alloys, quasicrystals, plastics, are used for various purposes or will be used in future. These materials have common nature that molecules or basic elements are arranged without spatial periodocity. Needless to say, properties of these materials depend much on the configuration of molecules (or elements) and basic understanding of such materials is possible only after characteristics of molecular configuration are known. Except for the quasicrystals, which have non-perodic but deterministic configuration of molecules, basic studies of these materials are made through statistical treatment of configurations, such as the tesselation of space, the packing of space or the network structure. This kind of studies belongs obviously to the science of pattern formation.

2.2 New trend in biological science

Creation of patterns in biological systems, i.e., the morphogenesis, is attracting interests of scientists in biology and medicine. This problem has long been persued merely by observing objects and describing what are observed qualitatively. However, recent development of computer has enabled us to study morphogenesis quantitatively.

On the other hand, several interesting experiments were made on the mechanism of differentiation by the use of hydrozoa, a primitive animal living in the fresh water (Gierer, 1975). The results suggest existence of a certain kind of "field" within this animal which determines appearances of organs at the early stage of development. The field is originally a mathematical concept and may be incorporated with the science of pattern formation.

2.3 Needs of pattern recognition

The pattern recognition itself is usually not an object of investigation but a tool for their porpose. In fact, most activities of pattern recognition are to develop techniques for respective problems. However, those techniques are closely related to a fundamental question in the science of pattern formation, i.e., what is the characteristic of patterns, how patterns are parametrized, how similarity of two patterns is defined, etc. Especially, a technology called "stereology" to reconstruct 3D structures from 2D images is becoming more and more important (Dehoff and Rhines, 1968; Miyamoto, 1991). An international organization, The International Society of Stereology, is established since 1960.

2.4 Mathematical analysis of social systems

Social systems, such as streets, railroads and public buildings, should be designed so that people can get the largest convenience. Here arises a problem of how the convenience is measured, how the distance of two regions is defined or how the center of a region is determined, etc. These problems are usually investigated in the fields of civil engineering or the social engineering. However, its basic concepts should belong to the science of pattern formation.

In accordance with the above recent situations research groups began to be organized in Japan. One was a group of people from various fields, mathematics, physics, chemistry, biology, geology, engineering, initiated by T. Ogawa (physicist) since 1980. They gathered together to exchange information and discuss on problems of pattern formation in respective fields. During this activity they found that they have a common base to make substantial discussion and even to make joint researches.

Another was a group of the stereology, where scientists from medicine, civil engineering, metallurgy and geology have been cooperating with an interest in reconstruction of 3D structures within human organs, alloys, rocks, and other composite materials. In 1960 The International Society for Stereology was established, and international cooperation has been maintained.

In 1984 these two groups began to cooperate with the initiative of S. Ishizaka (biophysicist), and in 1985 "The Society for Science on Form, Japan" was established. International symposia have been held in 1985 and 1988 (Ishizaka *et al.*, 1986; Ishizaka, 1989). The present author is now working as a president of this society. Most of our activity is interdisciprinary, and the society members have their own home fields. However, all of us have a hope that a new independent science will develop in future to investigate forms, patterns or morphological properties of objects in a basic manner.

As one of activities of this group a cooperative research project was undertaken in 1988 and 1989, supported by The Ministry of Education, Culture and Science and directed by the present author. Its results constitute the main part of this volume.

3. What Is Science of Pattern Formation?

The name of this science as a new kind of activity is not fixed yet. Even in this volume several names will be used, such as "science of pattern formation", "science of patterns", "science on form", "morphological science", etc. The present author would like to avoid discussion of what its name should be, but would like to explain what kind of activities are made within the scope of The Society for Science on Form, Japan. The activities are classified to the following three branches. This classification is applied also to the index of this volume.

3.1 Properties of space

This branch is concerned with statistical and geometrical studies of spatial division, packing, network, and other topological properties of objects. Needless to say, the geometry and the statistical geometry constitutes their basis.

3.2 Mechanism of pattern formation

In this branch mechanisms of pattern formation in physical, chemical, biological, geological and social systems are investigated. The tools are mainly deterministic mathematical principles, such as differential equation to predict future developments.

3.3 Method of pattern measurement

This branch consists of technologies to determine shapes or structures of objects from data obtained through camera, microscope, X-ray film, or sometimes boring of the earth. Obviously, the high-speed computer has contributed much to development of measurement techniques. The stereology mentioned above belongs to this branch, and the so-called information science is closely related to it.

The above three branches are keeping close cooperations with each other, and are directed to the final goal of understanding fundamental features of "Form". This notion is shown schematically in Fig. 6. The whole these activities are the "Science of Pattern Formation".

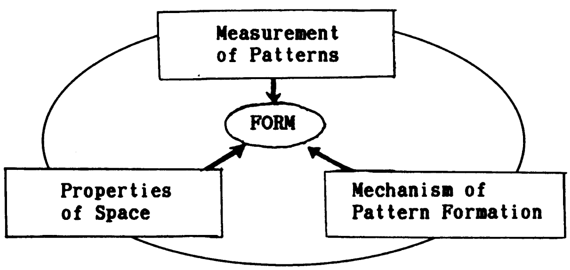

Fig. 6. Three branches of the science of pattern formation. They cooperate each other and are directed to a basic understanding of "Form".

4. Concluding Remarks

The present status of the science of pattern formation is an interdisciprinary field of activity, since it lies among many sciences mentioned in the previous sections. Then, does it hold this status for ever? Will it become an independent science? Or, will it be destined to disappear after technologies developed in this science are digested in other conventional sciences?

A hint for answering this question may be obtained by remembering the history of mathematics. The mathematical concepts of differentiation and integration were invented by Sir I. Newton and G. W. Leibniz in order to analyse motions of bodies. Note that Takakazu Seki, a Japanese mathematician of the 17-th century, established these mathematical concepts earlier than those Europian scientists, but they did not become popular because of the social situation of his era. Now, after the classical mechanics was established, did this mathematics disappear? No! On the contrary, it became an independent science and kept to be a central position in the modern mathematics. This example is very much suggestive.

Another example is also worth mentioning. The stereology, a branch in the third category of the science of pattern formation (see Section 3), is now being applied to many fields as a basic technology of measurement. Recently, an opinion is expressed frequently that the stereology should grow in future as a new science to give a bridge between "structure" and "function" of complex systems. This prospect is one of the important objects of the Science of Pattern formation.

Finally, it should be noted that we need an organization to connect scientists and a scientific journal in order to grow a science. In fact, The Society for Science on Form, Japan, and The International Society for Stereology are playing this role. The former is publishing a journal "FORMA", whose editing board includes both Japanese and foreign members, and the present author is working as the editor-in-chief. FORMA accepts any qualified paper, if it is produced based on the interest in "Form", irrespective of its object, method or result. The present author believes that the most effective way for promotion of the science of pattern formation is to enhance activities of these organizations and to have journals for publication of papers.

REFERENCES

Dehoff, R. T. and Rhines, F. N. (1968). Quantitative microscopy, McGraw-Hill.

Gierer, A. (1975). Hydra as a model for the development of biological form, Scientific American (Japanese edition, Feb. 1975).

Gransdorff, P. and Prigogine, I. (1971). Thermodynamic theory of structure, stability and fluctuations, Wiley-Interscience.

Gordon, R. and Jacobson, A. G. (1978). The shaping of tissues in embryos, Scientific American (Japanese edition, Aug. 1978).

Haken, H. (1981). Erfolksgeheimnisse der Natur-Synagetik, Deutsche-Verlags Anstalt.

Hirata, M. (1975). Patches on giraffe, Chuokoronsha (in Japanese).

Ishizaka, S. ed. (1989). Proceedings of the 2nd international symposium for science on form, KTK Scientific Publishers.

Ishizaka, S., Takaki, R., Kato, Y., and Toriwaki, J. ed. (1986). Proceedings of the 1st international symposium for science on form, KTK Scientific Publishers.

Kobayashi, T. (1961). Phil. Mag., **6**, 1363.

Manderbrot, B. B. (1977). Fractals—form, chance, and dimension, Freeman & Co.

Miike, H. (1988). Image processing study of Belousov-Zhabotinsky (BZ) system and Hydrodynamic phenomena in the chemical reaction, *FORMA*, **3**, 155–184 (in Japanese).

Miyamoto, K. (1991). This volume, pp. 00–00.

Taneda, T. (1988). Hydrodynamics via visual ization, Asakura-shoten (in Japanese).

Thom, R. (1975). Structural stability and morphogenesis, Benjamin Inc.

Thompson, D. W. (1969). On Growth and Form, revised by 1. Bonner, Cambridge Univ. Press.
Toda, M. (1981). In *Physics of random systems*, edited by The Physical Society of Japan, Baifukan (in Japanese).