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Map Fold a La Miura Style, Its Physical Characteristics and Application to the Space Science*

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Abstract. In this paper, the author presents a novel method of folding maps, its physical characteristics, and the similar method applied to the space science. Firstly, an introduction is given about a geometric surface which was discovered in the process of studying the mathematical theory of elasticity. Secondly, the proposed method of folding maps and its characteristics are described in detail. Finally, the application of the surface to the space structures, such as solar arrays and solar sails, are explained.

1. Introduction

Of maps that are inscribed on paper, the oldest one now existing is said to be one that represents a gold mine in the Nubian district, Egypt (Fig. 1). On the map, one can notice some parallel and equidistant lines extending lengthwise, apparently without any significant connection with the descriptions on the map. One may wonder what these lines mean.

It is possible that these lines are folds of the map, as can be inferred from their being parallel and equidistant. In any way, it seems certain that ancient people, like modern people, also attempted to fold up a map in various ways.

In the process of studying the mathematical theory of elasticity, the author discovered a peculiar geometrical surface (Miura, 1970; Tanizawa and Miura, 1978). Since this surface has an isometric nature that can be applied to a deployable

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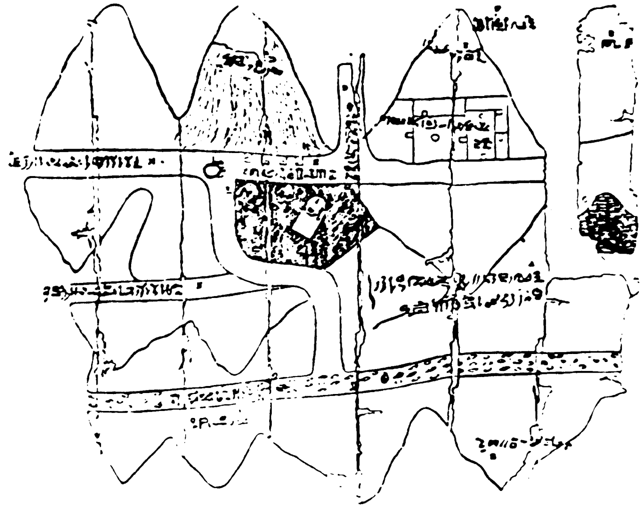

Fig. 1. Map of a gold mine in the Nubian district (a collection in Milano Museum, in the form of papyrus).

mechanism, I pursued the theoretical study aiming at such applications to packing large space structures like solar power arrays and space radars.

In this paper the author presents, as a practical application of this geometrical surface to map folding, a novel folding method essentially different from conventional ones, and detailed description of its features. In addition, the application of this surface to the space science is described briefly (Miura, 1978).

2. Geometry of Map Folding

By applying an external force, a sheet of paper can very easily be folded up or down, at any angle with respect to the surface of the paper. But the basic nature is that it hardly produces any stretch on its surface. Assuming an ideal paper that neither stretches nor contracts at all, then the process of paper folding can be described, in terms of geometry, as the isometric transfer of a plane. The isometric transfer is a transformation of a surface into another without producing stretch or contraction, and without resorting to cutting or conjoining. These conditions are the same as for both traditional paperworks (Origami) and maps. In the case of map folding, however, some additional conditions are required. A map should be folded in such a way that the transformed surface must also be a plane macroscopically, and have a smaller dimension, lengthwise or broadwise, than the original surface.

Within the knowledge of the author, it seems that there has been, from old times, fundamentally only one method of folding a map. In this method, a set of Cartesian coordinates (x, y) is appropriately assumed on the plane of the paper, then the paper is folded up in the x direction along folds parallel to the y axis, and in the y direction along folds parallel to the x axis. The order of folding can be chosen as desired. The paper must be deployed by following the same process in the reversed order. If a map of this folding is completely deployed into a plane sheet, it is found that the folds are two sets of parallel lines at right angles to each other, forming rectangular sections on the plane of paper. Thus, we will hereafter call this kind of folding "orthogonal folding".

3. Some Problems in Orthogonal Folding

We know that there are some problems in the orthogonal folding of a map. One of the problems is of human engineering nature. The action of folding and unfolding a map requires a considerably complicated motion of hands and fingers. In Fig. 2 is shown the loci, as recorded with an exposure camera, of motion of two thumbs in unfolding a map of orthogonal folding. The motion is, of course, more complicated in folding it. Complicated motion is inconvenient, particularly in the open air or in a limited space such as inside a car.

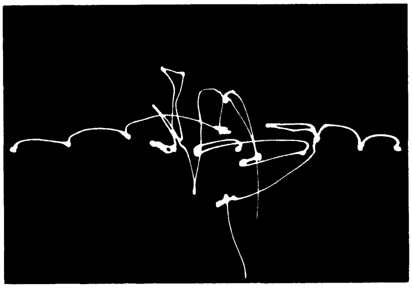

Fig. 2. Loci of two thumbs in unfolding a map of orthogonal folding.

Another problem is connected with what may be called stability of folds. Consider a folded map being unfolded flatwise. When the map is folded again, it frequently occurs that the concavity or convexity of a fold turns out the wrong way around, and the map can not be restored to the original state. It is for this reason that what may be called stability of folds is required in folding a map.

Finally, orthogonal folds place a lot of stress on the paper inducing, almost without exception, tears which begin where two folds intersect.

4. A Particular Polyhedral Surface and Its Application to Map Folding

There is a polyhedral surface having a very singular feature. It has been known among lovers of Origami, however, the exact characteristics of the surface has not been studied. The author re-discovered the surface, so to say, from the viewpoint of theory of elasticity and geometry (Fig. 3). It is called the developable double corrugation surface (DDC surface). Because only the geometric nature of this surface has bearing on our present problem, in what follows we see the surface only in this light.

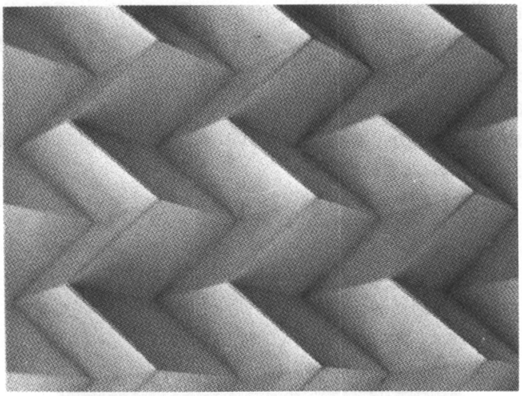

Fig. 3. Concave polyhedral surface of a particular nature.

In an accordion curtain, we may find an example of the most common way of folding up a plane in a fixed direction. Consider a sheet of ideal paper assumed to have no thickness. Folding up this paper infinitesimally closely will result in a straight line. In a word, this way of folding is equivalent to the transformation of a plane into a straight line. The orthogonal folding a map is, therefore, nothing but two successive transformations of this sort in two mutually perpendicular directions.

Is there a way in which a plane can be folded in two mutually perpendicular directions simultaneously? At first thought, one can hardly believe such a possibility, but the above-mentioned concave polyhedral surface does give a solution. In Fig. 4 there are a series of pictures showing how a plane is folded up and contracts itself lengthwise and broadwise simultaneously. The intermediate product of this process is a concave polyhedral surface consisting of a number of parallelograms. If it is folded up infinitesimally closely, then it will be folded up into a point. Conclusively, folding of this sort corresponds to transformation of a plane into a point.

A remarkable fact to be noted here is that the folds or contractions in two

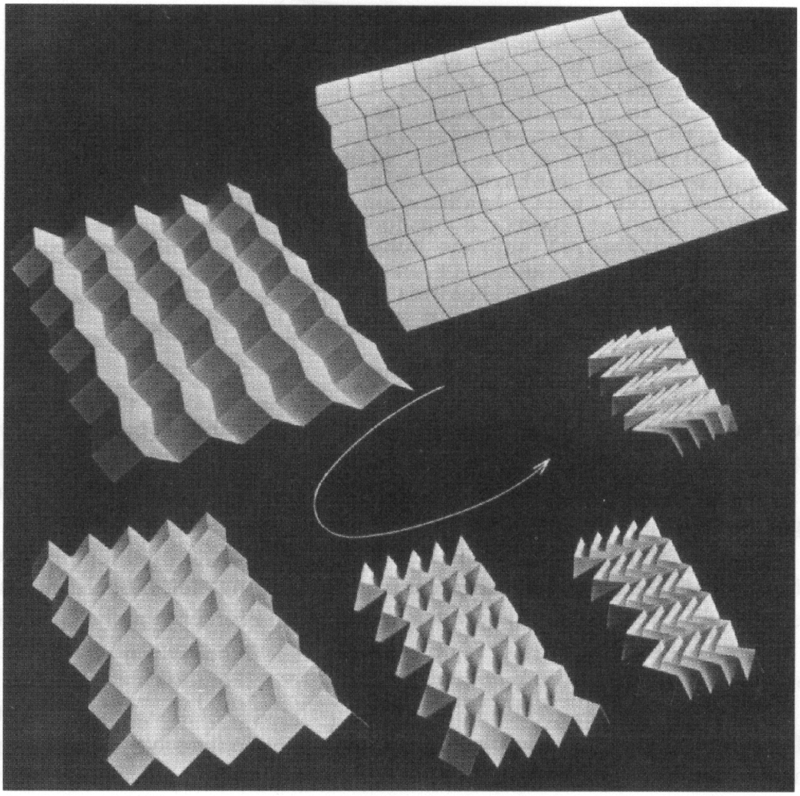

Fig. 4. Folding a plane in two directions.

mutually perpendicular directions are not independent to each other. The contraction in the x direction should always be accompanied by the contraction in the y direction, and vice versa. In contrast, the folding in the x and y directions are completely independent in the case of the orthogonal folding.

This particular DDC surface is composed of repetition of a minimum unit or a fundamental region. It is what composed of four congruent parallelograms as shown in Fig. 5. The whole surface is formed by parallel transformations in the x and y directions.

Consider a sheet of paper so folded that it forms a DDC surface, and see the motion of an arbitrary fundamental region. When a fundamental region is given some deformation, for instance, by making the fold angle sharper, the adjacent fundamental regions will undergo exactly the same deformation, which, in turn, causes further deformation to the fundamental regions adjacent to each of the deformed regions. Thus, in theory it is seen that the deformation given to a particular

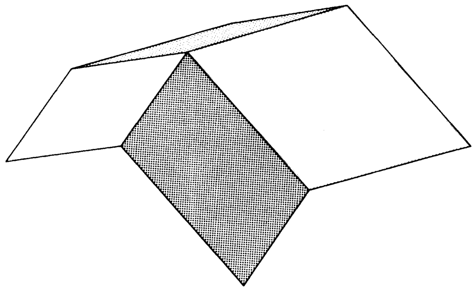

Fig. 5. Fundamental region of a concave polyhedral surface.

fundamental region will propagate over the entire surface in a second. A sheet of paper, when so folded, behaves as if it has a linkage mechanism built into it to fold or deploy itself. To fold it, it is necessary only to fold any one of the fundamental region.

In practice, the deformation in some part of the paper will not propagate to infinitely remote regions, but undergoes increasing attenuation with distance from the part at which the original deformation is initiated. In order to apply this method of folding to a practical map, it is necessary to make this attenuation as small as possible.

Generally, a map is rectangular in shape. To deploy a map folded up into a DDC surface, let us always pull the map at both ends of the diagonal of the rectangle. This will give an equal and simultaneous deformation to all the fundamental regions along the diagonal. Because these fundamental regions along the diagonal and those in their direct neighborhood cover the main part of the map, the desired deformation can be propagated, without decay, to most parts of the map.

In Fig. 6 there are a series of pictures showing the process of deployment of a map folded up in this way. Figure 7 shows the loci of two thumbs in the process of Fig. 6. Compared with the corresponding loci, shown in Fig. 2, for the case of the orthogonal folding, the movement of thumbs in this case is much smoother and simpler. Actual handling of the map is also so smooth that one feels as if the map folds and deploys itself automatically. Just a sheet of ordinary paper, if given these special folds, will move as if given a life.

5. Other Features of the New Method of Folding

By its inherent nature, the new method of folding gives an answer to the problem of fold stability discussed earlier. We assume that each fold has a sign of plus or minus depending on whether it is convex or concave, respectively. Consider what will happen when the sign of an arbitrary fold is reversed, for instance, a fold marked e in Fig. 8a changes from concave to convex. This will necessitate reversal

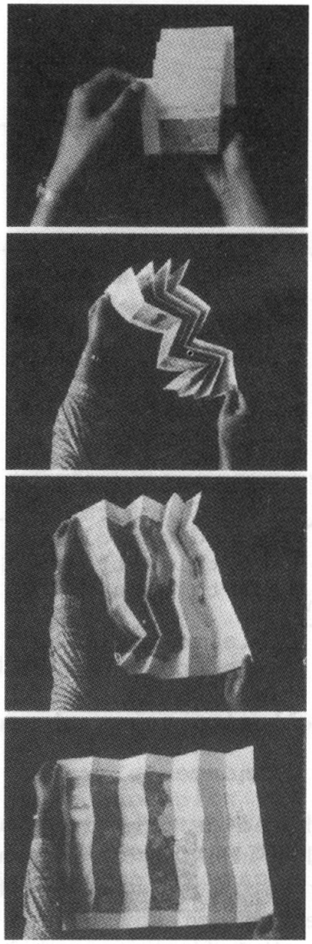

Fig. 6. Deployment of the new folded map.

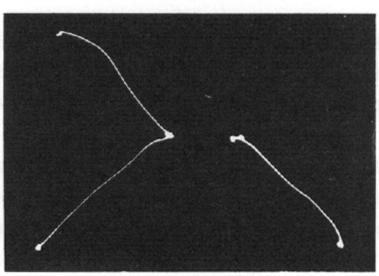

Fig. 7. Loci of two thumbs in unfolding the new folded map.

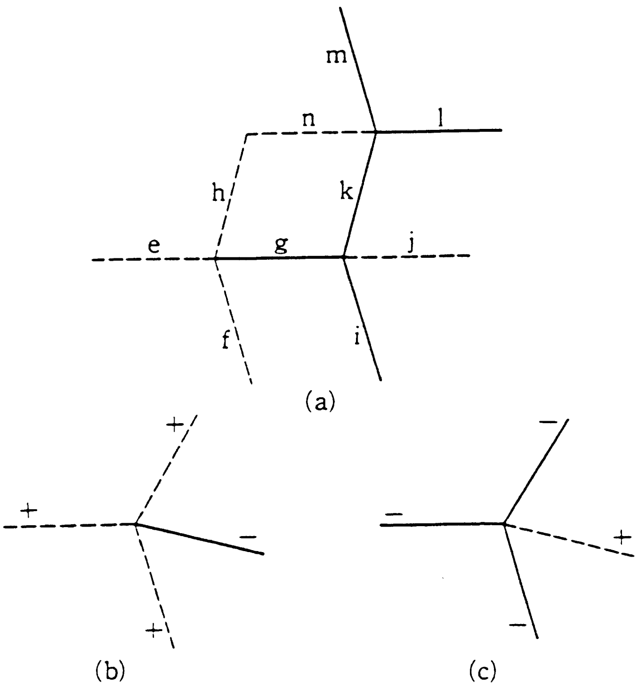

Fig. 8. Reversal of folding and the corresponding signs.

of all other folds, f, g, and h, having a nodal point in common with e. The reason for this simultaneous reversal lies in the fact that, when four folds have a nodal point in common, there are only two possible combinations of fold signs, as illustrated in Figs. 8b and 8c. The reversal of sign at fold g in Fig. 8a will in turn necessitate reversal of three folds, i, j, and k. In this way, reversal of an arbitrary fold necessarily leads to reversal of all the other folds on the map. Because such a situation should have a rare chance to occur, the initial fold remains unchanged. In other words, the fold is stable.

One can feel the truth of the above statement by actually manipulating a map folded up by this method. Moreover, one may feel as if there were a kind of memory built into the folded map, which precisely remembers all the signs originally assigned to the folds.

With regards to the strength of folds, the folded map under consideration has an essentially advantageous feature. In the case of the orthogonal folding, that portion of the paper which thrusts itself deeply inside the second fold causes a large tensile stress at the nodal point, whereas in the present case, only a single sheet of paper will come beneath the second fold, and with less depth. This helps reduce the above-mentioned tensile stress by a large margin. In addition, the inherent nature of immunity from fold reversal further contributed to the fold strength. As a result, the folds in our case is much stronger than those in the case of the orthogonal folding, and less vulnerable to breaking due to fatigue.

As a drawback to all these advantages, a weak point in our method of folding is that it can not be unfolded partially. This nature is inseparably related to the mechanism of deployment in our folding method and, therefore, unavoidable.

6. How to Fold a Map

Since our new folding is based on a form precisely determined by the geometric law, it will not start folding in a desultory manner. The basic step in design is to determine the parallelogram that will constitute the fundamental region, and then to assign positive or negative sign to its sides, or folds. There are three independent variables in a parallelogram. Other parameters are the initial size and the folded size of the map.

In order to determine these parameters, one must take into consideration the following factors:

1) Maneuverability in folding and unfolding
2) Thickness and stiffness and other characteristics of paper
3) Choice of the corner to be pulled out
4) Number of parallelograms in the directions of width and length (odd number is preferable)

As it is not the purpose of this paper to go into the details of design, let us restrict ourselves to a typical design, taking as example the widely used 1/50000 or 1/25000 topographical map issued by the Geographical Survey Institute of Japan (Fig. 9).

The first publication of the map in a form of our new design was made in corporation with Ollivetti Corporation of Japan and Godo Works Company. It is a map of Venezia designed by Jinnai and Watanabe, both are architects. We have developed an experimental "origami machine" for the purpose of folding maps. Attempts are made on folding maps by high speed automatic machines, however, it will need some time before developing a feasible mass production method.

Presently, the map fold in this style is called "Miura-ori", where "ori" is the Japanese word of "fold". There are a few proposed methods of folding it manually. Among them the method conceived by Professor Natori of Institute of Space and Astronautical Science is seemed to be the best. The illustration in Fig. 10 demonstrates his method applied to a sheet of standard size (A or B series) paper.

7. Application to Space Technology

Large planer membranes are mandatory for many space missions in the near future. Solar arrays, solar power satellites, solar sails, space radars are typical examples. Therefore, the technology necessary for the construction and packing of

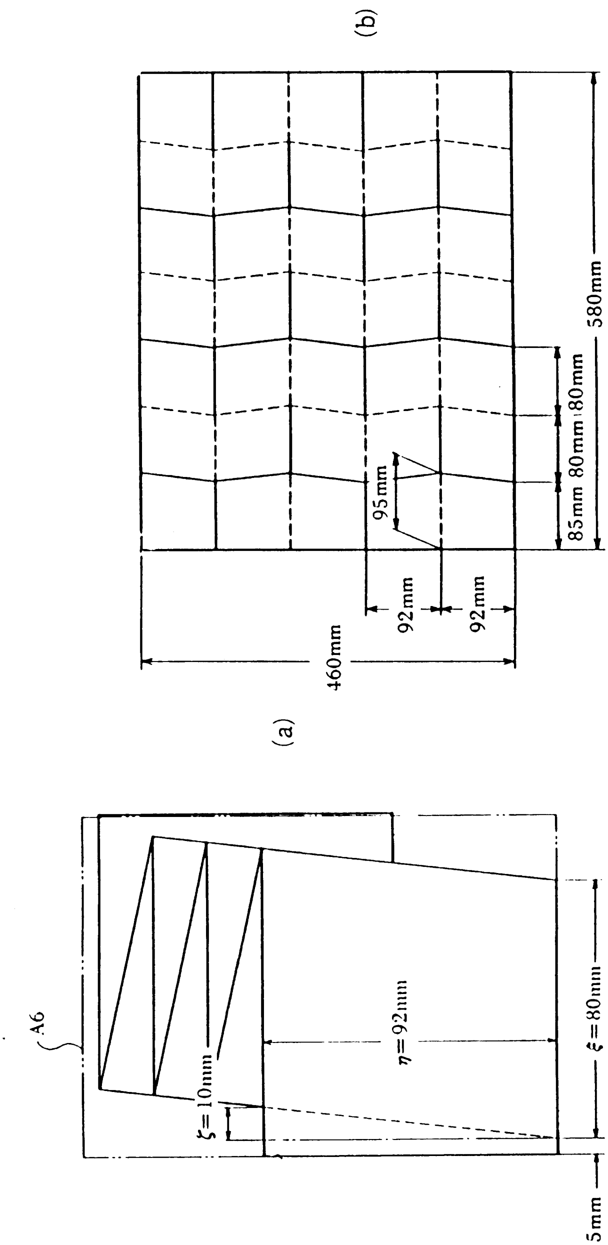

Fig. 9. A typical design example of the new folded map.

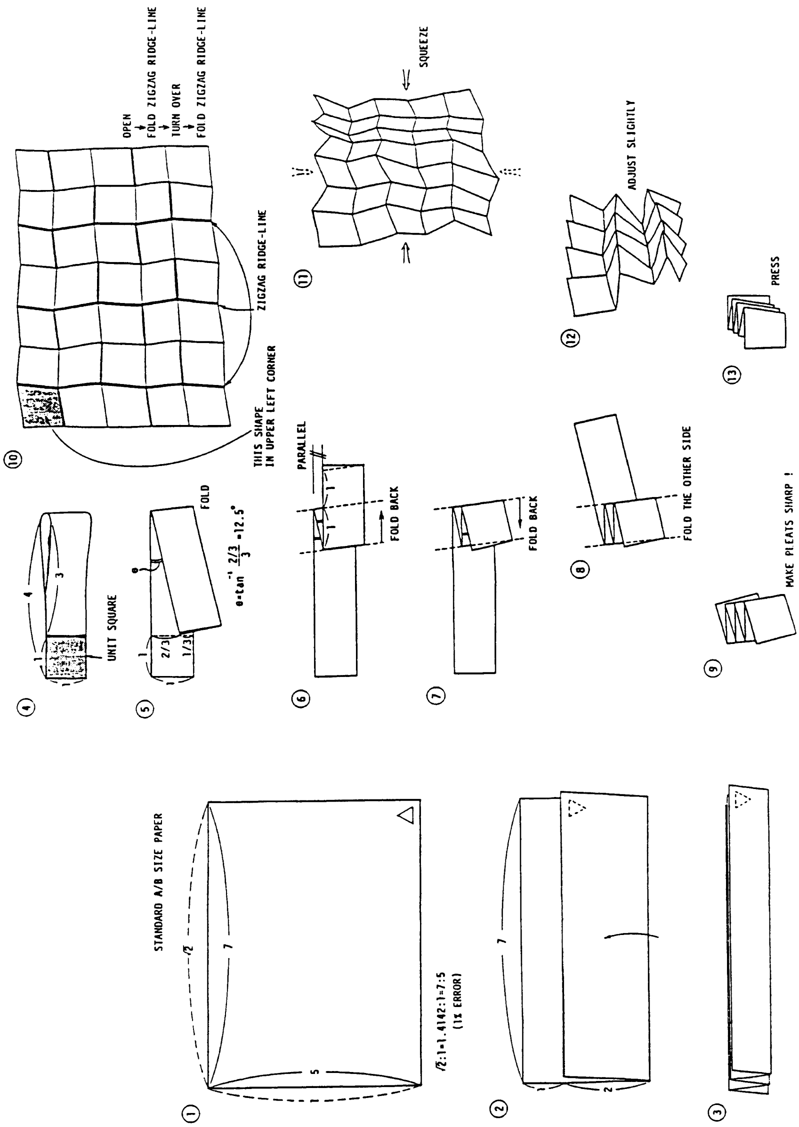

Fig. 10. How to fold Miura-ori, Natori's method (Illustration by M. Sakamaki).

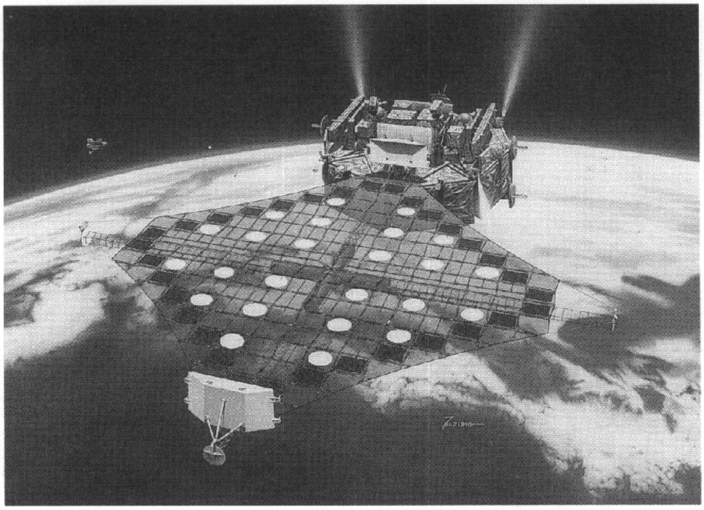

Fig. 11. 2-Dimensional deployable array experiment in space.

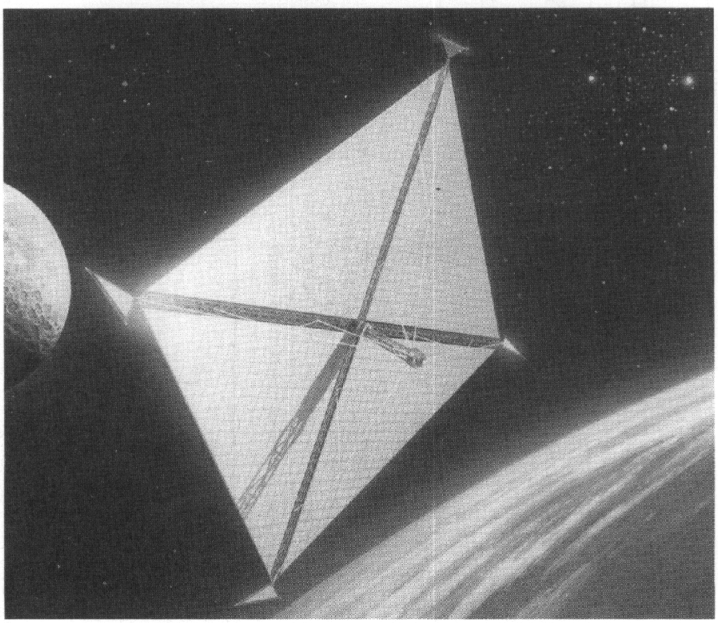

Fig. 12. Solar wail uses similar method of folding its wail.

these large membranes on the ground and their deployment in space must be established. The technology used for folding maps described in the previous sections was originally invented in order to cope with such problems of packing large membrane space structures (Miura, 1980).

For the purpose of testing its feasibility, the project is under way to launch a two-dimensional deployable array on board the space flyer unit (SFU) scheduled for 1994 (Fig. 11). It is a 6 × 6 meters square, thin membrane solar cell array which will be simultaneously deployed in two orthogonal directions. In this figure, one can observe that the fundamental unit is a parallelogram. Another application of this technology is the fabrication, packaging, and deployment of a huge sail made of very thin membrane for a solar sail spacecraft. The design of a solar sail spacecraft for the proposed moon race by the Solar Sail Union of Japan is based on this technology (Fig. 12).

8. Afterword

It is worthwhile, here, to add a few words about this new method of folding from different viewpoint. It is about the meaning of paper of this form as a printed media. Since the first publication of our research, a number of interested persons have contributed their comments on the subject. The most impressive among them is a notion that the map of this folding may give a new form to the "book" in a broad sense of the word, rather than being merely a map. As an attempt at realization, the author has recently published a catalog for the Exhibition "Koryo Miura-Structural

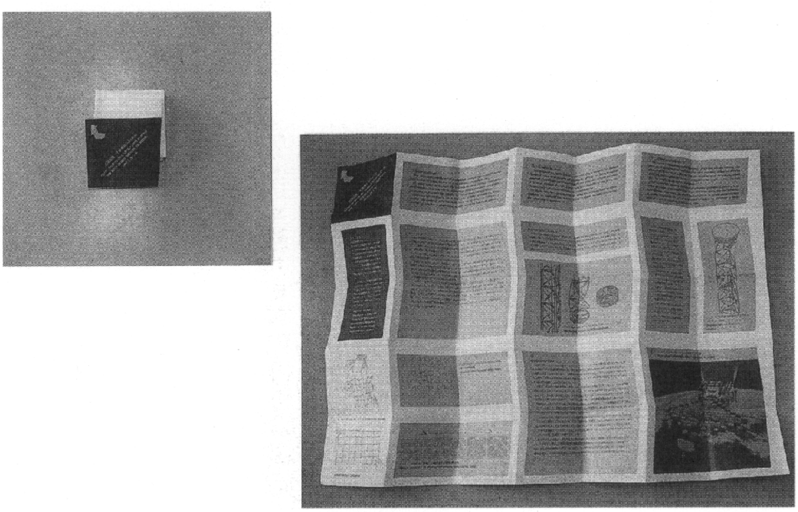

Fig. 13. A catalog for the exhibition in the new fold design.

Forms in Space" in this style (Fig. 13). This design gained general acceptance among the people.

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