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Dynamics of Random Interfaces with a Nonzero Initial Order Parameter

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The dynamic behavior of a system quenched below the order-disorder transition temperature is studied. The system consists of random interfaces between two stable phases and initially has a finite order parameter. It is found by the theory and the computer simulation that the area density of the interfaces shows an exponential decay with time, instead of the power law decay in the case of vanishing initial order parameter.

We will investigate the systems that the order parameter denoting the degree of the order is a scalar quantity and takes two values with opposite signs below the transition temperature, such as ferromagnets, binary alloys and so on. When the system is quenched from a high-temperature (disordered) phase into an ordered phase, distinct interfaces between the two stable states form. The progress of the system towards thermal equilibrium can be regarded as one in which the folded random interfaces shrink and gradually come loose. It is well known that the total area of the interfaces per unit volume decreases with time as $t^{-1/2}$ (Allen & Cahn:1979; Kawasaki & Ohta:1982). The initial state of the system can be supposed to be such that the interfaces are infinitely convoluted so that they fill up the whole system and

that two stable states occupy equal volume, that is, the initial order parameter M(0) vanishes.

In the present paper, we extend our study to the cases for nonzero initial order parameter and demonstrate that the time dependence of the area density A(t) shows an exponential decay, instead of the $t^{-1/2}$ law (Toyoki & Honda:1986). We illustrate the difference of the temporal behavior of the spin configurations in the ferromagnet between M(0)=0 and M(0)$\neq$0 cases in Fig. 1, which is obtained by a Monte Carlo simulation of a spin-flip kinetic Ising model (KIM). In the case M(0)=0, most of the sites belong to an infinite cluster, while in the M(0)$\neq$0 case some islands of the minor phase are surrounded by the sea of the major phase. The smaller the island is, the faster it shrinks. Thus it can be easily predicted that the case M(0)$\neq$0 results in the exponential decay of A(t). In the next section, we discuss quantitatively this statement by the Monte-Carlo simulation of KIM.

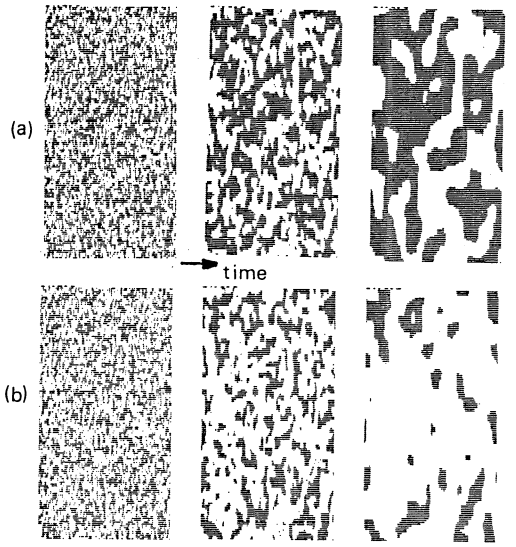

Fig.1 Temporal evolution of spin configurations: (a) M(0)=0, and (b) M(0)$\neq$0. Black (white) sites denote the up (down) spins.

NUMERICAL EXPERIMENT

The Hamiltonian for KIM is given by $H=-J\Sigma_{<i,j>}\sigma_i\sigma_j (J>0)$, where $\sigma_i=\pm 1$ and the sum runs over distinct nearest-neighbor pairs. The initial configurations are obtained so that spins on sites are randomly distributed under the fixed M(0). Our simulation is performed at zero temperature, because our main interest is not in the temperature dependence of A(t), but in their dependence on M(0). The system evolves by the standard spin-flip Monte Carlo procedure and the unit of time is estimated as the number of attempted spin flip per site. The spin flip is executed inevitably in the direction to make an energy lower for zero temperature. We simulate the systems of 135×135 spins in two dimensions and of 45×45×45 spins in three dimensions.

In Fig.2 we show various time (Monte-Carlo step) dependences of A(t), where A(t) is calculated as the number of bonds between neighboring spins with opposite signs, that is, $A(t)=-\Sigma(\sigma_i\sigma_j-1)/2$. The $t^{-1/2}$ behavior of A(t) is seen in the case M(0)=0. When $M(0)\neq 0$, A(t) decreases more rapidly than $t^{-1/2}$. The ratio of A(t) to $A_0(t)$, obtained from the initial condition of vanishing M(0), behaves as $\exp[-\Gamma t^{d/2}]$ [see Fig.3]. We find the dependence of Γ on the initial order parameter M(0) as shown in Fig.4, that is,

$$\Gamma \propto [M(0)]^{\delta} \qquad (1)$$

with the respective index $\delta=2.2$ in the two-dimensional case and $\delta=2.3$ in the three dimensional case. The error is estimated as about 10%, so that dependence of δ on the spacial dimension can not be decided on the basis of a few trials.

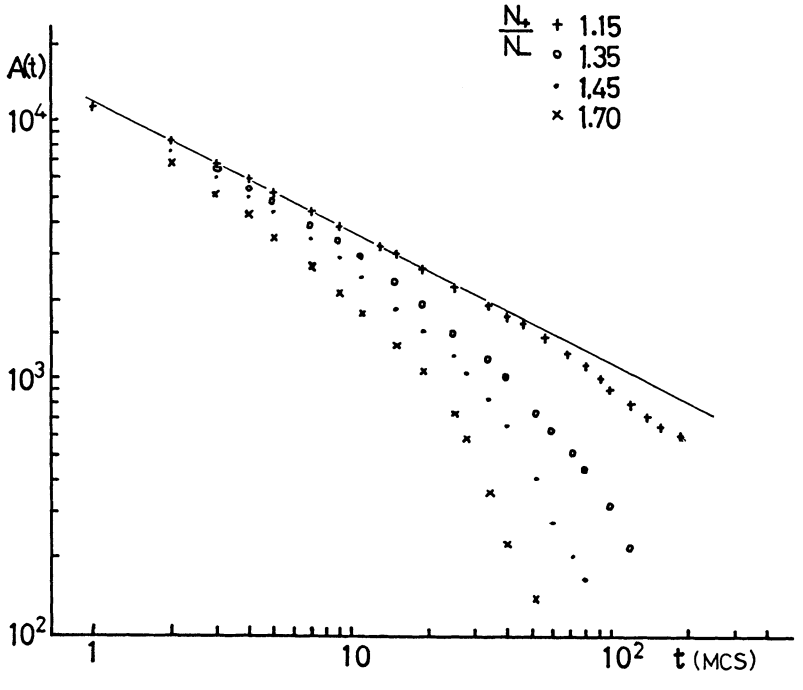

Fig. 2 Log-log plot of total length of interfaces versus time (in Monte Carlo steps) in the 135×135 system, where N_+ and N_- are the number of up spins and down spins at the initial state, respectively. Solid line indicates $t^{-1/2}$ decay.

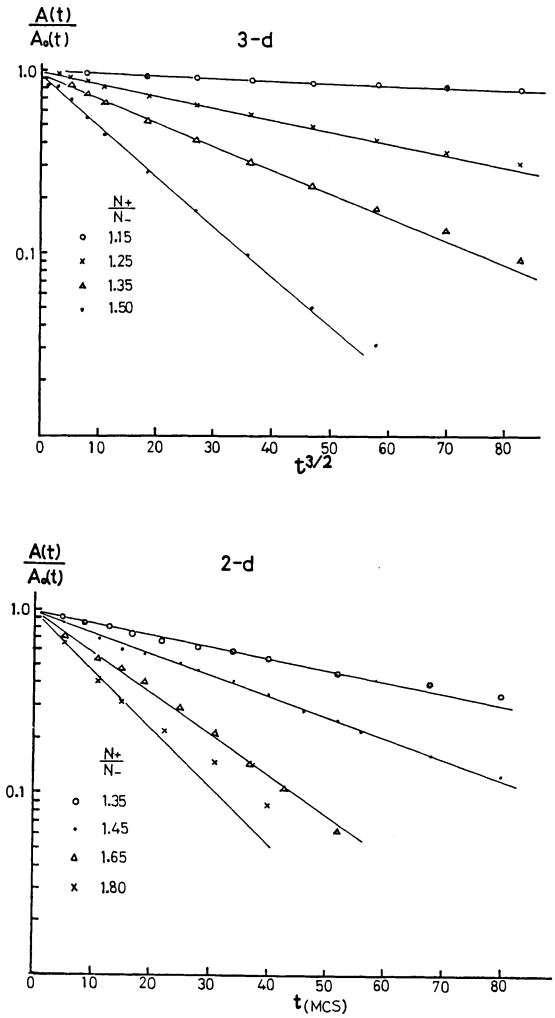

Fig.3 Semilog plot of $A(t)/A_0(t)$ vs $t^{d/2}$, where $A_0(t) \propto t^{-1/2}$. The spacial dimensionality is denoted by d. The upper figure is 45 ×45×45 system, and the lower one corresponds to 135×135 systems.

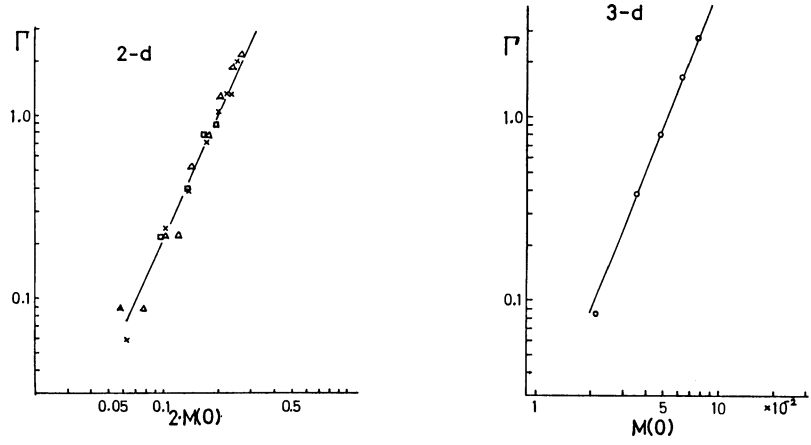

Fig.4 Log-Log plot of Γ defined in the text versus M(0). The factor Γ is evaluated from the slop in Fig.3. The straight lines have slopes δ=2.2 in the two dimensional system and δ=2.3 in the three dimensional system, respectively.

DEVELOPMENT OF THE u-FIELD THEORY

In this section we study analytically the motion of the random interfaces to examine the Monte-Carlo simulation results. The velocity normal to the interface at the position $\vec{r}(\vec{a},t)$ is given by (Allen & Cahn:1979)

$$v(\vec{r}) = L'K(\vec{r}), \tag{2}$$

where $K(\vec{r})$ is the local mean curvature and L' a kinetic coefficient. The position of the interface is a function of the

coordinate on the interface $\vec{a}$ and the time t. The theory by Ohta, Jasnow and Kawasaki (1982) seems to be most successful to investigate the dynamic of the random interface. Hereafter we will denote their theory u-field theory, after an artificial field introduced by them. Since in their work only the case $M(0)=0$ is considered, we develop the u-field theory to the general cases $M(0)\neq 0$.

In the u-field theory, the interface is represented by the nodes of a scalar field $u(\vec{r},t)$. The equation for $u(\vec{r},t)$ is determined so that the motion of its nodes should be equal to one derived from Eq.2. Since $du(\vec{r},t)/dt=0$ in the frame moving with the interface, the velocity of the interface is obtained as $v(\vec{r},t)=-(1/|\nabla u|)\partial u/\partial t$. Using the relation $K=\nabla\cdot\vec{n}$, where $\vec{n}$ is the unit normal vector given by $\vec{n}=\nabla u/|\nabla n|$, we have

$$\partial u/\partial t = L'(\nabla^2-\vec{n}\vec{n}:\nabla\nabla)u. \tag{3}$$

By neglecting the nonlinear terms, this equation becomes a diffusion equation:

$$\partial u/\partial t = L\ \nabla^2 u \tag{4}$$

with $L=L'(1-1/d)$. Averaging any quantity with respect to the configuration of the interfaces can be transformed to the one with respect to $\{u(\vec{r},t)\}$. The probability distribution of $\{u(\vec{r},t)\}$ is assumed naturally to be Gaussian with the mean value of $u(\vec{r},t)$, $<u(\vec{r},t)>=U$.

The density of the order parameter M(t) and the density of the area of interface A(t) are given , with use of a step function $\varepsilon(x)=\mathrm{sgn}(x)$, by

$$M(t) = V^{-1}\int d\vec{r}<\varepsilon(u(\vec{r},t))> \tag{5}$$

and

$$A(t) = V^{-1}\int d\vec{r}\int d^{d-1}\vec{a}<\delta(\vec{r}-\vec{r}(\vec{a},t))>$$

$$= <|\nabla u|\delta(u(\vec{r},t))>, \qquad (6)$$

respectively, where the angular brackets $<\cdots>$ denotes an averaged quantity over the probability distribution of $\{u(\vec{r},t)\}$. The Gaussian distribution of $\{u(\vec{r},t)\}$ immediately leads to

$$M(t) \propto \mathrm{erf}[x(t)], \qquad (7)$$

$$A(t) \propto (t+\alpha)^{-1/2}\exp[-x^2(t)], \qquad (8)$$

$$x(t) \propto U(t+\alpha)^{d/4} \qquad (9)$$

with the use of the correlation length α at the initial state. From (7), we have the physical meaning of U such as

$$U \sim M(0), \qquad (10)$$

where hereafter the small dependence of α is neglected. Substituting Eq.10 into Eqs.8 and 9, we obtain $\delta=2$ for the index in Eq.1, which differ from the value derived by the computer simulation. This may be arisen from the linear approximation for Eq.3.

To summarize, we conclude that the generalized u-field theory is successful in explaining the dynamic of the random interfaces with the arbitrary initial conditions.

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