Science on Form: Proceedings of the First International Symposium for Science on Form,
General Editor: S. Ishizaka, Editors: Y. Kato, R. Takaki, and J. Toriwaki, 103–111.

Phase Transitions in 2-dimensional Stochastic Cellular Automata

Yukito Iba, Yasuhiro Akutsu and Kunihiko Kaneko

Institute of Physics, University of Tokyo (KOMABA), Meguro-ku, Tokyo 153, Japan

Keywords: Cellular Automata, External Noise, Phase Transition

2-dimensional Cellular Automata are investigated under the external noise. Various kinds of interesting patterns are observed. For some rules, sharp phase transitions take place as the noise levels are changed.

INTRODUCTION

Cellular Automaton is a simple and concrete model to study the non-equilibrium phenomena and pattern formation. In spite of its extreme simplicity, it shows a surprisingly complex behavior and has a wide range of application. (Wolfram (1983),(1984), Packard and Wolfram (1985))

In this note, a class of the simplest 2-dimensional stochastic cellular automata are studied.

CELLULAR AUTOMATA

Cellular Automaton is a dynamical system on a lattice. On each site of the lattice, discrete variable S_i is assigned. S_i is updated synchronously by a simple "Rule".

For example, in "2-dimensional 5-neighbor" cellular automata, S_{ij} at t+1 step is determined by S_{ij} and $S_{i-1\ j}$, $S_{i+1\ j}$, $S_{i\ j+1}$, $S_{i\ j-1}$ at t step through a deterministic "Rule". (Fig 1)

A "Rule" can be represented in a "Rule table". (For example, Table 1)

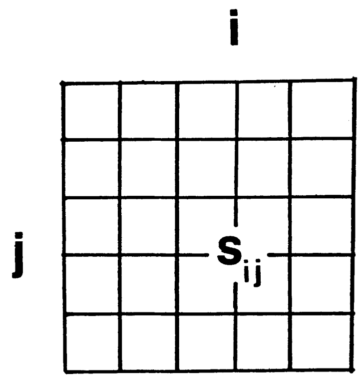
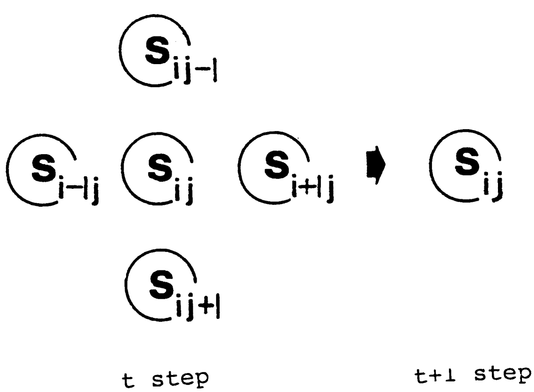


Fig 1

A five-neighbor cellular automaton and its dynamics.



----- Table 1 -----

	at t step				at t+1 step
S_{ij}	$S_{i-1\ j}$	$S_{i+1\ j}$	$S_{i\ j-1}$	$S_{i\ j+1}$	S_{ij}
0	0	0	0	0	1
0	0	0	0	1	1
0	0	0	1	0	0
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	1	0
0	0	1	1	0	1
	* * * * * *				* * *
1	1	1	1	1	1

An example of "Rule table" for general "5-neighbor" rules.

STOCHASTIC CELLULAR AUTOMATA

There are many ways to introduce the "stochasticity" into cellular automata .(Schulman and Seiden (1978) , Grassberger et al. (1983) , Kinzel (1985) , Grinstein et al. (1985))

In our model, the simplest type of external noise is used. Each variable S_{ij} changes its value with probability **p** after each update. (**p**:external noise level)

Cellular automata can have a huge number of attractors. The external noise brings about jumping among the attractors and a small number of stationary states are selected.

OUR MODEL

The simplest 2-dimensional cellular automaton is a "5-neighbor" one.(See, **Fig 1**) It is very hard ,however,to study all possible rules even for this case.

Here, we restrict ourselves to the rules which satisfy the following two conditions.

1.OUTER TOTALISTIC:
S_{ij} at t+1 step depends only on itself and the sum of nearest neighbors ($S_{i-1,j} + S_{i+1,j} + S_{i,j-1} + S_{i,j+1}$) at t step.

2.SYMMETRIC:
Symmetric under the (0-1) transformation.
(S_{ij} <-> $1-S_{ij}$)

For "Outer totalistic" rules, the "Rule table" can be expressed in the simpler form in **Table 2**. If a rule is also symmetric, (**) in **Table 2** is determined by the symmetry from the upper lines (*),hence (*) can be used as a "code" of the rule.

The number of rules which satify 1 and 2 are 2^5 = 32. Not all of them, however, are independent. There are two transformations which relate one rule to another rule . (Kaneko and Akutsu (1986)) For example, the rule 11001 (**Fig 3**) is equivalent to the rule 10011 ("Roll",**Fig 2**) under "Ferro-Antiferro" transformation.

Thus, only 10 rules remain independent. These "primary" rules are investigated here.

----- **Table 2** -----

at t step		at t+1 step	
S_{ij}	$S_{i-1\ j}+S_{i+1\ j}+S_{i\ j-1}+S_{i\ j+1}$	S_{ij}	
0	0	1	
0	1	1	
0	2	0	(*) :**CODE**
0	3	0	
0	4	1	
1	0	0	
1	1	1	
1	2	1	(**)
1	3	0	
1	4	0	

An example of "Rule table" for "outer totalistic" rules.

RESULTS OF THE SIMULATION

Results are summarized in Table 3 . For some rules, "phase transitions" can be observed as the noise levels are increased. For other rules, there are no phase transitions and patterns gradually change as the noise levels. Patterns generated by the rules are given in Fig 3.

Details of the results are presented elsewhere. (Kaneko and Akutsu 1986 , Kaneko,Akutsu and Iba 1986)

Table 3

code	pattern	phase transition	remark
00000	trivial	no	
01010	turbulent	no	"additive"
00010	ferro	yes	*)
00011	ferro	yes	
00101	ferro	yes	
00100	ferro+antiferro	yes(1st order)	**)
10011	roll	yes(1st order)	
10001	labyrinth	no	
00001	glassy	no (?)	
10010	glassy roll	?	

*) " Ferro " means a uniform pattern . Two different uniform patterns (all 0 or all 1) are equally possible at the low-noise-level phase.

**) " Antiferro " means the checkerboard-like pattern. " Ferro+Antiferro " means four patterns - two Ferro and two Antiferro - are equally possible at the low-level-noise phase.

ACKNOWLEDGEMENTS

We would like to thank Dr.S.Wolfram for fruitful discussions. We also thank to Prof.T.Izuyama for useful discussions and critical reading of the manuscript.
We thank Institute of Plasma Physics at Nagoya for the facilities of FACOM M200.

TYPICAL INITIAL CONDITION

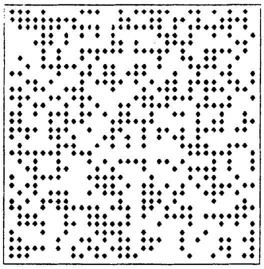


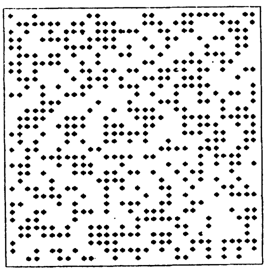
Fig 2

Pattern generated by each rule.
For each rule, one high-noise-level pattern and two low-noise-level patterns at different Monte Carlo steps are shown.

initial condition : random start
p : noise level
mcs : Monte Carlo steps

TURBULENCE 01010

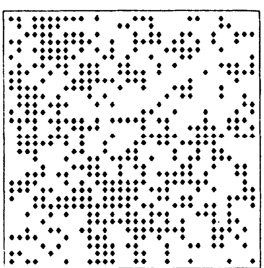
P=0.0001



MCS=1500

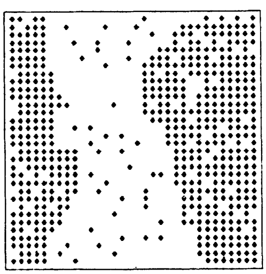
FERRO 00010

P=0.2

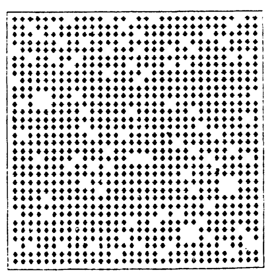


MCS=1500

P=0.01



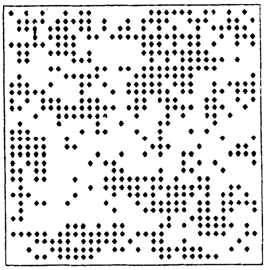
MCS=1500



MCS=20000

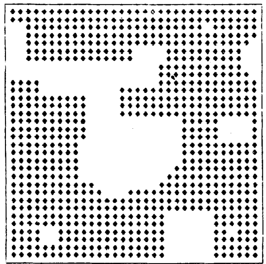
FERRO 00011

P=0.2

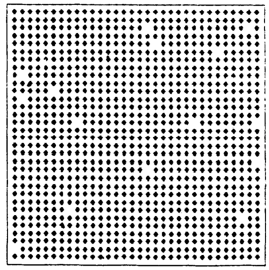


MCS=1500

P=0.01



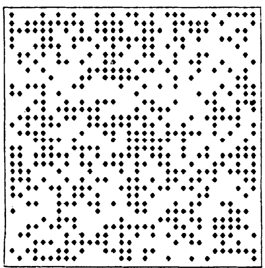
MCS=250



MCS=8000

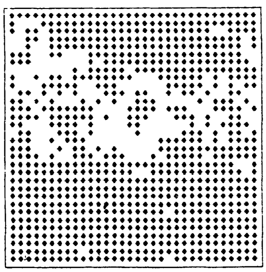
FERRO 00101

P=0.2

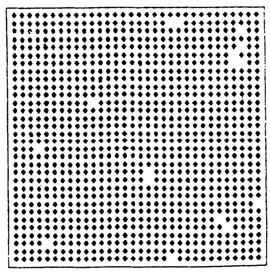


MCS=1500

P=0.005



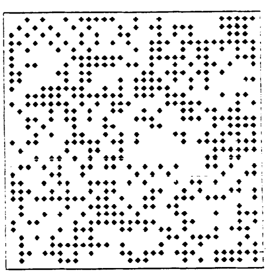
MCS=250



MCS=1500

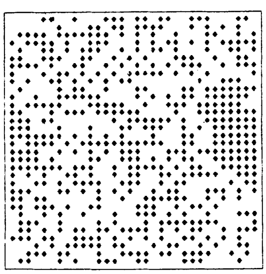
FERRO+ANTIFERRO 00100

P=0.005

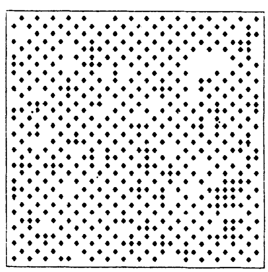


MCS=5000

P=0.0001



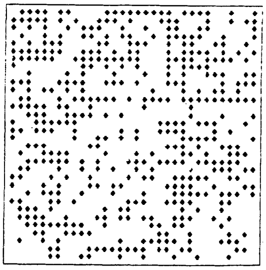
MCS=1500



MCS=12000

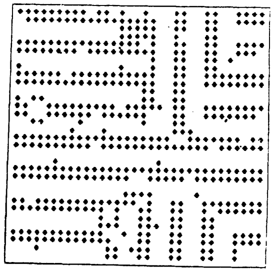
ROLL 10011

P=0.05

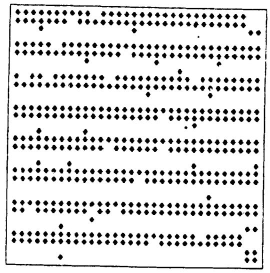


MCS=1500

P=0.001



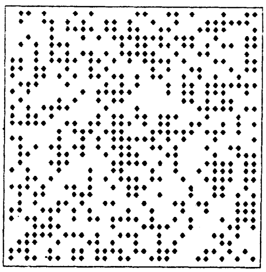
MCS=1500



MCS=5000

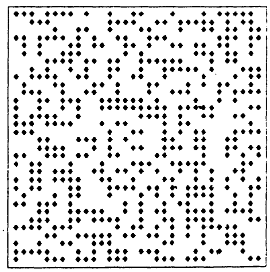
LABYRINTH 10001

P=0.2

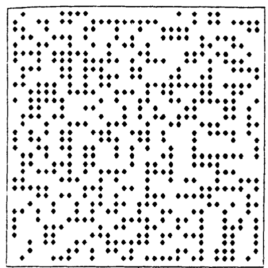


MCS=1500

P=0.001



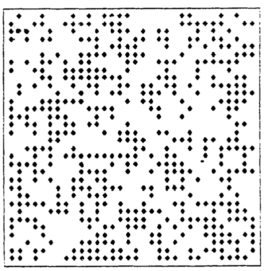
MCS=1500



MCS=5000

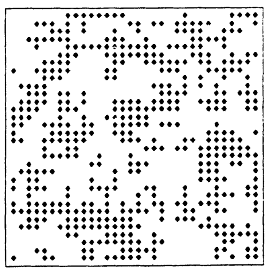
GLASSY 00001

P=0.2

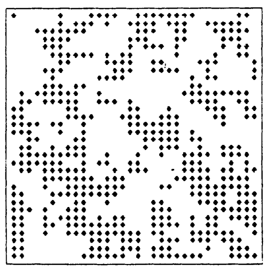


MCS=1500

P=0.001



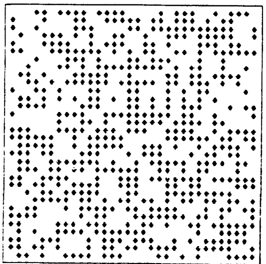
MCS=1500



MCS=5000

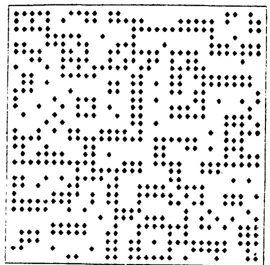
GLASSY ROLL 10010

P=0.05

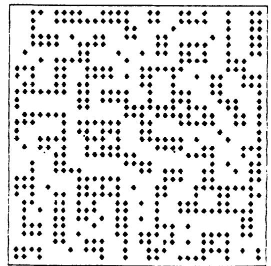


MCS=1500

P=0.0001



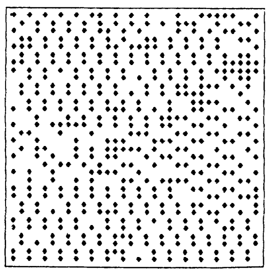
MCS=5000



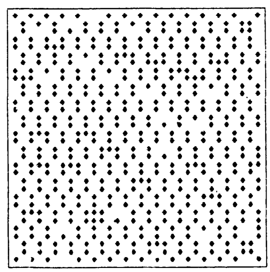
MCS=20000

Fig 3
An example of the **NON** "primary" rule. The rule here is equivalent to "Roll 10011" under "Ferro-Antiferro" transformation.

P=0.001



MCS=1500



MCS=5000

REFERENCES

Grassberger P, Krause F, Twer T.v.d, (1984): J.of Phys **A17** L105
Grinstein G, Jayaprakash C, He Yu, (1985): Phys.Rev.Lett **55** 2527
Kaneko K, Akutsu Y, (1986): J.of Phys A in press
Kaneko K, Akutsu Y, Iba Y, (1986): in preparation
Kinzel W, (1985): Z.Phys. **B58** 229
Packard N, Wolfram S, (1985): J.Stat.Phys. **38** 901
Schulman L S, Seiden P E, (1978): J.Stat.Phys **19** 293
Wolfram S, (1983): Rev.Mod.Phys. **55** 601
Wolfram S, (1984): Physica **10D** 1

2-10

C: Patterns by Cellular Automata Modeling are similar to Turbulent flow patterns. The value 1 or 0 corresponds to Turbulent or Laminor regions. It is advisory to compare it with Turbulent flows in Boundary layers. (R. Takaki)