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Size, Shape and Texture Analysis

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Morphological Analysis has developed from empirical beginnings through phenomenological studies and is now moving on a more theorectical path along with applications. A guiding principle behind this development has been the necessity to take into account the high level human ability to qualitatively assess size, shape and texture of objects. For this reason it has always been an important aim of the research to develop methods of analysis which would yield morphic descriptors for size, shape and texture that would enable the regeneration of the image of the object. In turn this has facilitated efforts to allocate physical interpretations to the various morphic descriptors obtained. This is invaluable in connection with model building and also with the understanding of mechanisms.

I. Morphological Analysis

Many developments in science start out in life as one or a series of empirical observations. In turn these lead investigators to probe into the unknown in order to see if these empirical observations have a wider significance. This stage of development of a field may be termed phenomenological. Sooner or later a third stage of development occurs in which basic theoretical structures are formulated and developed. Finally application occurs and a new technology becomes available for all who wish to use it. The evolutionary history of a new technology is not always as cleanly compartmentalized and the development of Morphological Analysis is a good example to illustrate this (1).

In general, the classical methods for the measurement of particle shape were known or suspected to be ambiguous, and this state of affairs promoted the experimental work in morphological anlaysis at the beginning of the 1970's. A major impetus occurred when the National Science Foundation supported an International Workshop on Morphological Analysis at the University of Iowa in 1977 (2). Since that time a great deal of phoenomenological research has been conducted. The 1980's have seen three important developments. The start of a major theoretical work, the availability of a new instrument for conducting morphological analysis (Shape Analyzer^{TM}) and the beginning of industrial applications of morphological analysis.

During this present century in Physics, there has been a steady movement away from the micro-structure, towards at one end of the scale the extremely large, as in the study of astronomical phenomena, and at the other end of the scale to the extremely small, as in the study of subatomic particles. This abandonment of what might be called the middle world, that is the realm of size between the microscopic and the sub-microscopic, has been most obvious in the areas of materials and particle technology. Because it just so happens that the micro-structural features of materials and the particles of particle technology are of microscopic dimensions. Morphological analysis was originally developed as a means of measuring the shape of small particles. With the advent of texture analysis is has now become clear that it is possible not only to measure the size and shape of small particles but also to measure the structure characteristics of these systems, including the internal structure and surface structures of particles as well as the structure of particulate assemblies. However, recent theoretical developments (as yet unpublished) indicate that morphological analysis has potentially wider application than was first conceived for it. Theoretical work is now underway to develop analytical solutions to a variety of problems including Stokes sedimentation, flow through irregular pipes, and numerous other problems in fluid dynamics, which have hitherto escaped analytical solution.

To summarize therefore, morphological analysis appears to be gathering momentum and it's range of applicability is widening. These include:

- Particle Technology
- Micro-structure Analysis
- Fluid Dynamics

II. Requirements of Analysis Method

It is desirable and indeed necessary that the morphological descriptors of size, shape and texture should have certain attributes. These include:

- The morphological descriptors are supported by sound theoretical basis and are rigorously defined.
- They are invariant.
- They are treatable with normal statistical methods.
- They are capable of physical interpretation.
- They can be used to regenerate the original size, shape and texture features.
- A suitable instrument should be available to carry out these measurements and associated calculations.

III. Definitions and Theoretical Basis

Experimental work in Morphological Analysis was begun in the late 1960's. And on a purely trial and error basis the (R,θ)Method was developed. In those days progress was very slow because of the lack of suitable equipment.

Another obstacle to the further evolutiion of morphological analysis was the lack of a definition of shape. The early work focused on the measurement of the shape of profiles and therefore a definition of the shape of a profile was sought. A thorough search of the literature failed to reveal any definition whatsoever. This literature reviewed revealed that investigators invariably set off to develo a method for measuring what they perceived to be the shape of an object, without actually ever having defined the term for shape which they were trying to measure. It took an embarrassingly long time to develop an operational definition of shape (in this case the definition of the shape of a particle profile). An acceptable definition seemd to be, "the pattern of all the points of the surface or profile of the particle." Once this definition was in place the further evolution of morphological analysis began to speed up.

When it came to the measurement of texture, a similar situation obtained. Fortunately it was possible to analog the definition of shape and develop an operational definition of texture, "the pattern of distribution of gray levels within the profile of the particle." (3). The sense of these two definitions for shape and texture is that in effect shape deals with the characteristics of the frame of the picture and the texture deals with the characteristics of the picture itself. Clearly in the case of texture the inference to be drawn depends upon the physics of the process by which the image was obtained. Thus the gray levels can mean a variety of things from difference in chemical composition to a difference in height, to a difference in the arrangement of identical objects, and so on.

The calculus of variations has proved to be a most useful tool in the development of the morphological analysis theory. For example using this calculus it has been possible to develop the (R,θ)Method theoretically and thus confirm what had previously been developed empirically. This is done by maintaining the area of the particle profile constant and allowing the perimeter to vary in order to introduce shape into the theory. This success prompted the use of the calculus of variations to develop relationships expressing the two and three dimensional morphic characteristics particles. This important work is being carried out by D. W. Luerkens, and the three Luerkens Equations are shown in Table 1 (4). The $R(\theta)$ Equation is a Fourier Equation. The $G(r,\theta)$Equation is combination Fourier and Bessel. The $R(\theta,\psi)$Equation is combination Fourier and Legendre Polynomials. The corresponding coefficients are included in these three equations.

Key - $R(\theta)$: describes the particle profile.
$G(r,\theta)$: describes the gray level of the surface.
$R(\theta,\psi)$: describes the surface of a solid.

Dimension	Equation	Application
1-D	$R(\theta)= a_0+\sum_n a_n \cos n\theta + b_n \sin n\theta$	Shape Analysis...
2-D	$G(r,\theta)=\sum_n\sum_j (a_{nj}\cos n\theta + b_{nj}\sin n\theta)J_j(r)$	Surfaces, Texture...
3-D	$R(\theta,\varphi)=\sum_n\sum_j (a_{nj}\cos n\theta + b_{nj}\sin n\theta)P_j^n(\varphi)$	Particle Behavior...

TABLE 1

THE LUERKENS EQUATIONS

IV. Theoretical Development

A. Size and Shape Morphological Descriptors (5)

The (X,Y) profile pairs obtained from the digitized image are converted to polar coordinates in terms of (R θ) pairs, with the center of gravity of the profile taken as the geometric center. The (R θ) pairs are expanded into a Fourier series, the coefficients of which are formulated into the Fourier descriptors. The R(θ) relationship may be written as follows:

$$R(\theta) = a_0 + \sum_{n=1}^{N} (a_n \cos n\theta + b_n \sin n\theta) \qquad (1)$$

In which:

$R(\theta)$ is the radius at angle θ, the a_n's and b_n's are coefficients, n is the harmonic number, and a_0 is the mean radius.

To make the eventual Fourier descriptor rotationally invariant, the coefficients in the above equations are size normalized and modified:

$$L_0 = \frac{a_0}{R_0} \qquad (2)$$

In which:

L_0 is the size normalized mean radius.

$$L1(n) = 0 \text{ for all } n$$

$$L2(n) = \frac{1}{2R_0^2}(a_n^2 + b_n^2) \qquad (3)$$

In which:

L2(n) is the size normalized sum of the squares.

$$L3(m,n) = \frac{3}{4R_0^3}(a_m a_n a_{m-n} - b_m b_n a_{m+n} + a_m b_n b_{m+n} + b_m a_n b_{m+n}) \qquad (4)$$

In which:

L3(m,n) is the size normalized sum and difference multiples.

R_o is the "equivalent radius," i.e. the radius of a circle with the same area as that of the particle profile and

$$R_o = \sqrt{a_o^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)} \qquad (5)$$

The above size and shape terms are related to the mean and the moments about the mean of the profile radial distribution. A summary of the relationships may be written as follows:

$$\mu o = \quad L_o R_o \text{ (mean radius)} \qquad (6)$$

$$\mu 1 = \quad R_o \sum_{n=1}^{\infty} L1 (n) = 0$$

$$\mu 2 = \quad R_o^2 \sum_{n=1}^{\infty} L2 (n) \qquad (7)$$

$$\mu 3 = \quad R_o^3 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} L3(m\ n) \qquad (8)$$

In which:

μ_o is the mean of the radial distribution.

μ_1, μ_2, μ_3 are the first, second and third moments about the mean of the radial distribution respectively. The morphological features which are used to illustrate applications of morphological analysis to particle technology problems include the following:

R_o, the <u>size</u> term, where πR_o^2 is the area of the profile.

The <u>not-roundness</u> of the profile which is the radical standard deviation of the shape.

L_2 (2), indicates the <u>elongation</u> of the profile.

L_2 (3), indicates the <u>triangularity</u> of the profile.

Ruff, indicates the roughness of the profile and is $\sum_{n=5}^{N} L_2$ (n)

An additional morphic feature is the rotational symmetry defined as: (4)

$$C_m = \frac{\Sigma\ L_2\ (m\ n)}{\Sigma\ L_2\ (n)} \quad \text{in which } m=1,\ 2,\ 3, \ldots\ldots M \qquad (9)$$

Other symmetry operations include reflection, rotation-reflection and inversion. Some of these are summarized below:

Inversion

$$I = \frac{\Sigma\ L_2(2n)}{\Sigma\ L_2(n)} \qquad (10)$$

Rotation-Reflection

$$S_2 = \frac{\Sigma L_2(2n)}{\Sigma L_2(n)} \quad (\sigma_1 \& \sigma_4) \qquad (11)$$

B. Texture Morphological Descriptors (3)

The particle image will yield information the nature of which is dependent upon the physics of the situation. For example, the field of view may be of a rough surface. In this case the gray level differences will indicate surface altitude differences. Alternatively, the features of the image may be caused by chemical variations within the particle, in which case, the gray level variations will indicate the microstructure of the particle. An assemblage of particles can be represented as a pattern of pixels of different gray levels and analyzed accordingly.

The texture morphological descriptors that have been developed thus far are of three kinds. Statistical properties of the gray level distribution, symmetry operations on a total image, and textural features. Statistical properties permit convenient generalization of the image. Symmetry operations indicate the extent to which the texture possess rotation reflection and inversion symmetries. The texture morphology features themselves can be used to regenerate the original image. Therefore it is to be expected that specific textural morphic features will indicate recognizable characteristics of the texture. It will be possible therefore to develop the correspondence between mathematical textural descriptors and verbal textural descriptors.

1. Statistical Properties

$$\bar{G}_T = \sum_{i=1}^{L} i\ Pi \qquad (12)$$

$$\sigma_T^2 = \sum_{i=1}^{L} (1 - \bar{G}_T)^2\ Pi \qquad (13)$$

In which:

$\bar{G}_T$ is the mean gray level of the image.

σ_T is the standard deviation of the image gray level.

L is the maximum gray level.

Pi is the probability distribution at gray level i.

2. Symmetry Properties

If the image is rotated by $^{2\pi}/n$ and the original configuration is reproduced, the image is said to possess an n-fold rotational symmetry.

The textural symmetry is:

$$Cn = 1 - \frac{\iint [G(x,y) - TG(x',y')]^2\, dxdy}{\iint G(xy)^2\, dxdy} \qquad (14)$$

(TDs)

In which:
$G(xy)$ is the gray level at (x,y)
$x'y'$ are locations corresponding $G(x,y)$

3. Textural Features

The Fourier-Bessel combined coefficients have the form:

$$A_{nm} \qquad B_{nm} \tag{15}$$

In which n + m are the harmonic number in the r +θ direction respectively.
The physical indications of these Fourier-Bessel Descriptors have yet to be elucidated.

V. The Shape Analyzer^{TM}

The Shape Analyzer^{TM} is a scientific instrument which is used to analyze the shape, size and texture of objects. the item for analysis may be the object itself, its photograph, optical or electron micrograph, etc. The shape-size analysis is achieved by converting the profile to a set of shape and size descriptors which are complete, unequivocal and invariant. The texture analysis converts the full image of the object into a set of textural descriptors which are also unequivocal, complete and invariant. These shape, size and texture descriptors can be used in research, in quality control, and in process control and specifications.

The Shape Analyzer^{TM} consists of a high quality graphics system and integrated software. Associated peripherals and ample memory space are provided. The instrument can be used in the laboratory and also in on-line, real time applications. The instrument is shown in Figure 1. A sample menu is given in Table 2.

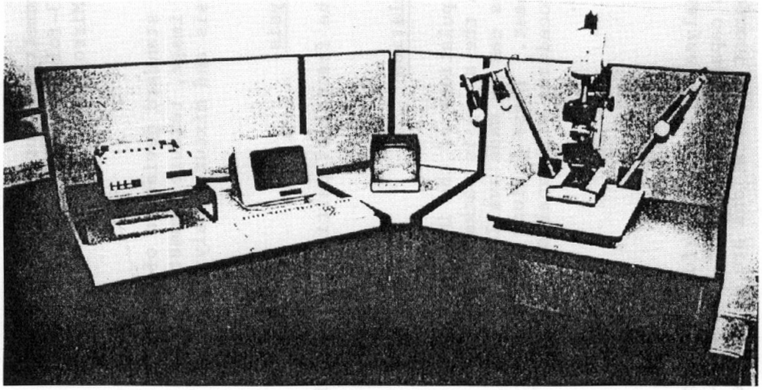


FIGURE 1 - Shape Analyzer^{TM} by Shape Technology Limited

I. Classical Measurements of Particles

A. Size
Equivalent radius: mean equivalent radius, standard deviation, distribution curve.

B. Classical Measures
Perimeter, Hausner shape factors, Martins diameter, Feret diameter, two dimensional sphericity, rugosity.

II. Morphological Analysis

A. Shape
Not-roundness, elongation, triangularity, squareness, roughness. Mean and standard deviation of these shape features, distribution curve. Shape composition (R,Θ), $(\phi,1)$ and (R,S) methods of analysis.

B. Interior Measures Include:
Porosity - total area, relative area, pore size distribution, mean and standard deviation of pore size. Pore shape: mean, standard deviation and distribution of shape features selected.

C. Shape Symmetry Measures
2-fold, 3-fold, 4-fold rotational symmetry. Mirror symmetry and inversion.

D. Texture
Mean and standard deviation of gray level of image, texture symmetry, color analysis and mixture analysis.

III. Fractal Analysis

Measure of the fractal of particles selected.

IV. Image Manipulation

Various manipulations of the image are available in the analysis package and additional ones can be made available by special request. These include image enhancement procedures with analysis and edge removal.

V. Versatility

The Shape Analyzer^{TM} has a powerful computing and graphics capability for which varied in-house development applications are available.

TABLE 2

MENU OF SHAPE ANALYZER^{TM}

VI. Results

A. Theory

The theoretical developments discussed in Section IV. of this paper have been accompanied by a more specialized method of analysis. For example, in the case of particle shape, it is true that there is a general class of particles that may be described as $R(\theta)$ particles but there also exist particles with re-entrant profiles, such that for a particular value of theta, R will have more than one value. The (ϕ,ℓ) Method was developed to cope with this problem, however the mathematics are not as pretty as those of the R-Theta Method and it has not found such general use (6). A general method for all classes of particles is available termed the R,S Method (7). However, at the time of writing this analysis technique has not been used in any application.

B. Instrumentation

The development of a sophisticated state of the art instrument like the Shape Analyzer™ should not preclude from consideration alternative methods of carrying out the digitization portion of the analysis. Methods that have been used and still can be used include: digitization by hand, the use of a manually operated crosswire, the use of a digitizing or graphics board, the use of a proprietary tracing system called a graph pen, the link up of a TV camera graphics boards and suitable computer.

C. Shape Classifiers

There have been numerous studies of the use of morphological analysis to classify particulate materials, these are summarized in Table 3.

Material	Features	Reference #
Metal Powders	$\sum_{n=1}^{n=n_1} An$ vs $\sum_{n=n_1}^{n=n_2} An$	8
Metal Powders	envelope statistics of powder signature	9
Grain Dust	various	10
Dislocation Networks in C.W. Brass	C_6 vs $L_{2(2)}$	11
Fusain & Vitrain	$L_{2(2)}$ vs L_0	12
Freeze Dried Coffees	C_2 vs C_3	13
Blocky & Agglomerate Al_2O_3	$L_{2(4)}$ vs $\sum_{n=1}^{N} L_{2(n)}$	14

TABLE 3
SHAPE MORPHOLOGY CLASSIFIERS

D. Sampling

Some recent work reported in the literature indicates that special care has to be taken in sampling particulate material for morphological analysis because of the small size of the eventual sample that is analyzed (this may be as small as 100 particles) (15). However other work as yet unpublished, shows that the variation from sample tos ample is very small and that the sampling method used in the study did not affect the results significantly (16). It should be noted that although samples as small as 100 particles can be routinely tested in morphological analysis, there is no practical upper limit to the number of particles analyzed, except for the fact that it takes more time to do the extra ones. A study carried out some years ago in our laboratory showed that there was no appreciable difference between the results obtained from a sample of 1000 particles, 750, 500, 400, 250, 100, 80 75 and 50 particles per sample respectively. This analysis was carried out using the R-Theta Method it does not imply that the same sort of results would be obtained for all classes of particulate material and for the other methods of morphological analysis.

E. Particle Formation Studies

Morphological analysis lends itself to the study of modes of particl formation. This sort of investigation can lead to a deeper understanding of the mechanisms of particle formation. An important practical results will be the development of the ability to manufacture particulate materials of a desired morphology. It has been shown for example that single step processes of particle formation such as atomization, comminution and crystallization tend to produce prototype morphologies (17). On the other hand studies in which materials have been processed with varied sequences of processes tend to indicate that the morphology of the progeny particles are most strongly influenced by one dominent process in the sequence (16). An important application of morphological analysis of particles occurs in the filed of wear debris analysis (18). Work going on in this area is developing the capability to identify and differentiate wear mechanisms by study of the morphology of the debris resulting therefrom. Work is also underway to enable the observer to identify the chronological stage which the wear process has reached (19). Attrition of particulate solids is a significant problem in many applications. Some preliminary work on the attrition of coffee freeze dried products has been conducted (13).

F. Particle Behavior

The importance of particle morphology upon the packing and flow behavior of dry particulate solids is well known. By conducting flow experiments on monomorphic sets of particles of regular geometry, a three-dimensional shape correlation is developed as shown in Figure 2 (20). This indicates that the correlation may be used as a starting point, to enable

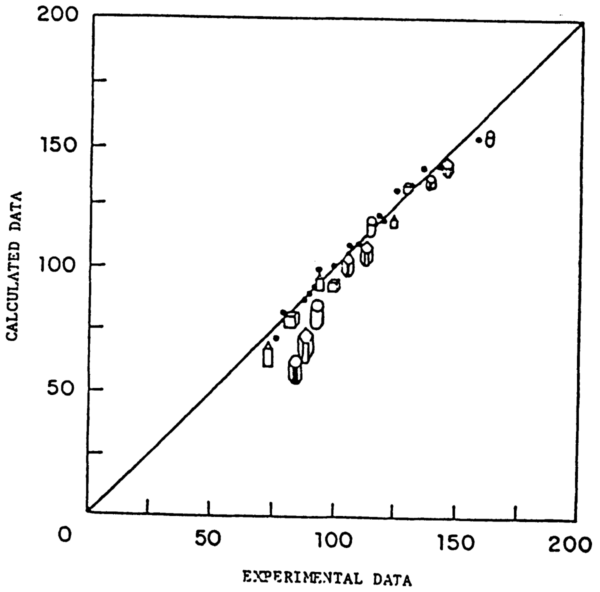


FIGURE 2

investigators to predict the effect of particle shape variation upon dry solids flow behavior.

Sphericity is often quoted and used in the literature to denote an indicator of particle shape. The three symbolic particles shown in Figure 3 demonstrate that there is some difficulty with the concept of sphericity as a good indicator of shape. All three particles in Figure 3 have the same sphericity but they clearly have totally different shapes. These considerations have lead to several investigations being conducted in order to demonstrate the relative utility of morphological shape descriptors over the use of sphericity. These are conducted in the phenomena of sedimentation (21, 22), slurry flow (23), and fluidized bed behavior (24). In the case of the sedimentation analysis, it was demonstrated that three dimensional morphological shape correlation was highly accurate and even enabled the observer to differentiate both on the basis of shape and also orientation of the sedimenting particles. In the case of the slurry flow and fluidized bed investigations, it was demonstrated that in the case of particles with identical values of sphericity but different shapes the behavior was indeed affected by the particle shape.

Morphological analysis has been shown to be useful in following the progression of the chemical reaction (25) (zinc dissolving in HCL). More recently the technique is being used to study the effect of particle size and shape distributions upon dust explosibility. The effect of particle size on dust explosibility has been well publisized. But the effect of particle shape is less clearly understook, as indicated in the case of

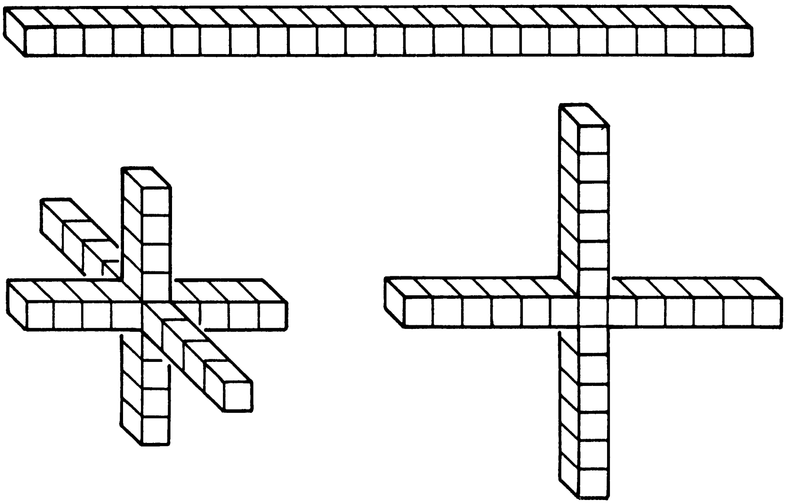


FIGURE 3

aluminum powders in Figure 4 (28). Only in the last few years has it been found possible to observe the effect of abrasive particle shape, size and roughness upon the abrasive wear rate of materials. These interesting results have shown for example that the effect of particle shape upon abrasive wear rate may be dominant (26). That increased particle roughness produces greater material wear rate and that the particles themselves undergo size, shape and roughness changes during the wear process (27). They are in effect machined during the process.

G. Texture Morphology Analysis

Texture Morphology Analysis makes it much easier to analyze mixtures of particulate solids. Up to five component systems have been analyzed. Some results for the two component mixture study are shown in Figure 5. It is a very interesting result: two different methods have been used to analyze the mixture. The texture morphology analysis has produced the texture standard deviation and also the two, three and four-fold texture symmetry's. The other method used was a simple counting routine in which black/white or white/black crossover's were counted and recorded. It is believed that the junction of the three symmetry curves in Figure 5 indicate that the system is fully mixed at that point in time. The analysis of micro-structures has long been an objective of morphological analysis, some preliminary work is being carried out using morphic size and shape descriptors, in the case of grain, size and shape, shapes of dislocation networks and effects of cold rolling on micro-structure (11). Texture morphology analysis per se has not yet been used, as at the present time the development of the textural morphic features method is not complete.

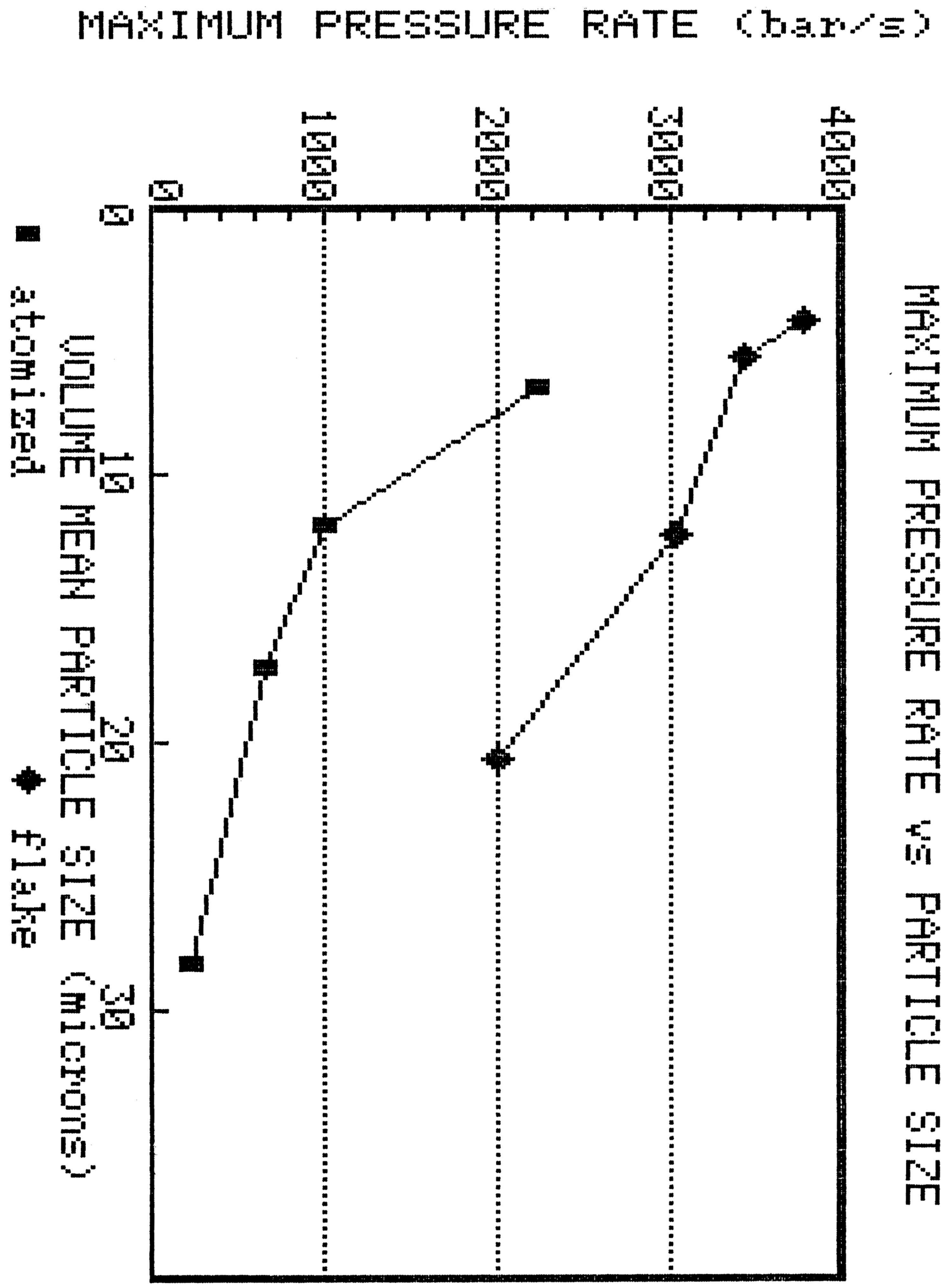
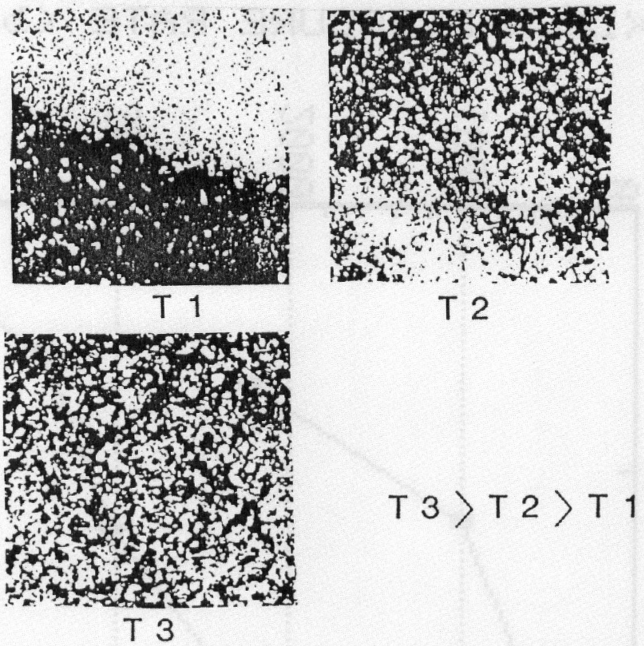

MAXIMUM PRESSURE RATE vs PARTICLE SIZE
MAXIMUM PRESSURE RATE (bar/s)
0
1000
2000
3000
4000
0
10
20
30
VOLUME MEAN PARTICLE SIZE (microns)
■ atomized
✦ flake

FIGURE 4

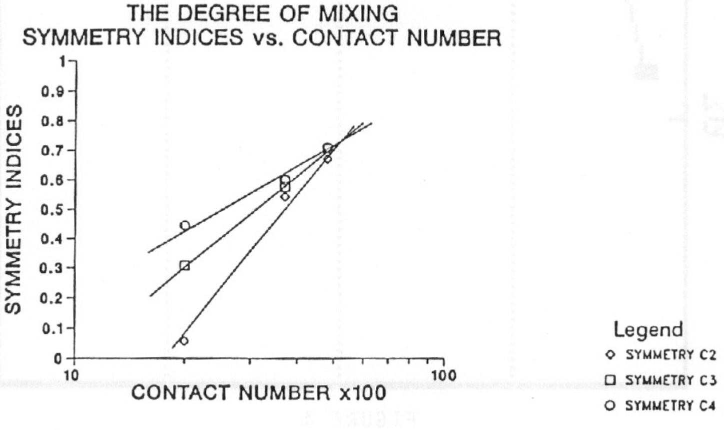


MIXING

Mixture of equal parts of black and white particles after times T_1, T_2, & T_3

TEXTURE SYMMETRY

2-fold, 3-fold and 4-fold rotational symmetry of the texture of the image.



The convergence point indicates complete mixing.

FIGURE 5

VII. Future Developments

Future developments in size, shape and texture analysis are many and include, a general theory of characterization, a completion of the theoretical development of texture morphology analysis and the gaining of understanding of the physical interpretation of the descriptors, development of introduction of new instruments for the 3-D characterization of particulate materials based on different physical principles then those of the Shape Analyzer^{TM}. A broader acceptance and application of morphological analysis including not only general engineering applications such as heat transfer, flow in ducts and porous media and the like, but general applications of the analytical techniques in other areas of science and engineering.

VIII. Conclusion

Morphological Analysis has developed from empirical beginnings through phenomenological studies and is now moving on a more theoretical path along with applications. A guiding principle behind this development has been the necessity to take into account the high level human ability to qualitatively assess size, shape and texture of objects. For this reason it has always been an important aim of the research to develop methods of analysis which would yield morphic descriptors for size, shape and texture that would enable the regeneration of the image of the object. In turn, this has facilitated efforts to allocate physical interpretations to the various morphic descriptors obtained. This is invaluable in connection with model building and also with the understanding of mechanisms.

IX. References

1. Beddow, J.K., Particulate Science and Technology, Chemical Publishing Company, New York, 1980. See also "Particle Characterization in Technology," ed. by J. K. Beddow, CRC Press, Boca Raton, Florida, 1984.
2. Beddow, J.K., and Meloy T.P., editors, Advanced Particle Morphology, CRC Press, Boca Raton, Florida, 1980.
3. Hsyung, Niey-Bor, MS Thesis, "Particle Texture Analysis," University of Iowa, 1984.
4. Luerkens, D.W., to be submitted to Particulate Science and Technology, an International Journal.
5. Luerkens, D.W., Beddow, J.K., and Vetter, A.F., "Invariant Fourier Descriptors," Powder Technology, 31, 2, 217-220, 1982.
6. Fong, S.T., Beddow, J.K., and Vetter, A.F., "A Refined Method for Particle Shape Representation," Powder Technology, 23, 17-21, 1979.
7. Luerkens, D.W., Beddow, J.K., and Vetter, A.F., "Generalized (RS) Method of Morphological Analysis," Powder Technology, 31, 2, 217-220, 1982.
8. Beddow, J.K., Philip, G.C., Vetter, A.F., and Nasta, M.D., "On Relating Some Particle Profile Characteristics to the Profile Fourier Coefficients." Powder Technology, 18, 1977, pp. 42-44.
9. Singh, P., Characterization of Powders," Ph.D. Thesis, IIT Bombay, Department of Metallurgical Engineering.
10. Beddow, J.K., Vetter, A.F., and Musgrove, D., "Morphic Characteristics of Grain Dust," in Particulate Science and Technology, 325-334, 1983.
11. Beddow, J.K., "On Potential Application of Morphological Analysis in Metallography," in Powder Metallurgy and Selected High Temperature Materials, edited by P. Ramakrishnan, Oxford and Delhi, Bombay, 29-46, 1985.
12. Beddow, J.K., Vetter, A.F., and Hansen P., "Morphological Analysis of Selected Coal Macerals," CIC Conference, Nova Scotia, 87-90, June 1981.
13. Beddow, J.K., et al., to be published.
14. Hsieh, H.P., "Morphological Analysis of Alumina and its Trihydrite," AIChE, Paper 26, New York, 1985.
15. Hawkins, A.E., Davies, K.W., "Sampling Powders for Shape Descriptors," Particle Characterization, 21-27, 1984.
16. Chen, Y-L, M.S. Thesis, University of Iowa, 1984.
17. Guo, A., Beddow, J.K., and Vetter, A.F. "Particle Morphology in Relation to Manufacturing Methods," Powder Metallurgy International, 1, 27-29, 1983.
18. Beddow, J.K., Fong, S.F., and Vetter, A.F. "Morphological Analysis of Metallic Wear Debris," Wear, 58, 201-211, 1980.
19. Knapp, J. work in progress.
20. Guo, A., Beddow, J.K., and Vetter, A.F., "A Simple Relationship Between Particle Shape Effects and Density, Flow Rate, and Hausner Ratio," Powder Technology, 43, 279-284,1985.

21. Ah-Chin, A., Beddow, J.K., and Vetter, A.F., "Sedimentation Experiment," submitted to Powder Technology.
22. Carmichael, G.R., "Estimation of the Drag Coefficient fo Regularly Shaped Particles i Slow Flows from Morphological Descriptors," Ind. Eng. Chem., Process Des. Dev., 21, 401-403, 1982.
23. Yin, M.J., Beddow, J.K., and Vetter, A.F., "Effects of Particle Shape and Two-Phase Flow in Pipes," submitted to Powder Technology.
24. Rajpal, S., Beddow, J.K. and Vetter A.F., "Effects of Particle Shape on Particle-Fluid Behavior," (a) "The Scope of Particle Shape Quantifiers in Fluidization Technology." Powder and Bulk Solids Conference, Atlanta, GA, 578-595, May 24-26, 1983.
25. Fong, S.T., Beddow, J.K. and Vetter, A.F., "Acid-Zinc Reaction Predictably Affects Particle Morphology," Powder Technology, 23, 1979.
26. Vetter, A.F., and Swanson, P.A., "Particle Morphology Applied to Characterizing Abrasive Materials," Particulate Science and Technology, 1, 2, 127-138, 1983.
27. Vetter, A.F. and Swanson, P.A., "Morphological Analysis Applied to Abrasion Studies," Proceedings of the Int'l. Bulk Solids Conference, Atlanta, GA, 638-648, 1983.
28. Ogle, R.A., Beddow, J.K., Vetter, A.F., and Chen, L-D., "The Effect of Particle Size and Shape on Dust Explosion Intensity," Particulate Science and Technology, (in preparation).

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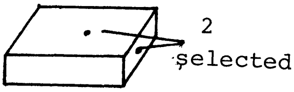
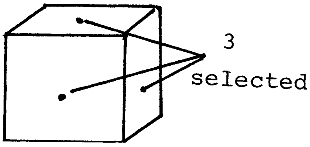
Q: I am very much impressed by such a rapid development of morphological analysis of particulate matters. But it must also be pointed out that the applications of their results are still very limited. This is mainly due to the fact that the analysis has still remained at the stage of two dimensional one. Therefore my question is whether your methods described in your paper can be extended to three dimensional analysis or not. I would also expect some comments on the future prospect of the techniques of three dimensional morphological analysis. (G. Jimbo)

A:

1. If the particle shapes are isotropic, profile analysis is satisfactory + produces significant results.
2. If the particle shape is anisotropic, it is nessesary to intelligently select the profiles:
   ** example **





3. Please refer to my paper for the Luerkens Equation:
   Profile - Fourier
   Texture - Fourier-Bessel
   3-D - Fourier Legendre
4. We are now developing the instrument for 3-D.
5. Luerkens is working out the cases of the general and special fibres.

Q: You showed us a few picturees showing degree of mixing of textures (or others). Is thereanyquantitative method characterizing the degree of mixing? (R. Takaki)

A: In the example : When the texture symmetries are equal, the binary system is fully mixed.

$TS_2 = TS_3 = TS_4$

This is also true for 3,4 and 5 component mixes.