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Adjacency Relations among Figures on a Digitized Image Plane with Applications to Texture Analysis

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In this paper, we present methods to describe neighboring relations among connected components scattered randomly or regularly in a two dimensional digitized image. Three types of adjacency relations; modified digital Voronoi neighbor (MDVN), modified digital relative neighbor (MDRN) and modified digital Gabriel neighbor (MDGN), are defined by extending the VN, RN, and GN to a set of connected components in a digitized image. The derived adjacency graphs are used to analyze textural properties of an input image. Algorithms to obtain the above adjacency graph are briefly presented with evaluation of the amount of computation. Several possible applications are suggested with experimental results.

INTRODUCTION

Various methods for shape analysis of digital pictures have been developed (Rosenfeld & Kak:1982). However most of them are concerned with shape features of figures and few methods have been developed for analyzing features in the way of arrangement of figures.

Neighborhood relations of a point set on the continuous space and tessellation of space based on their relations have attracted growing attention in the field of computational geometry recently (Toussaint:1980, Toussaint:1985).

In this paper, we aimed to develop a new feature analysis technique of digital pictures by introducing those concepts into digital picture processing. For the purpose, we extended three neighborhood relations of a point set – Voronoi neighbor, relative neighbor and Gabriel neighbor to a connected component set on a digital picture plane and developed algorithms to obtain those relations actually. Those extended neighborhood relations proved to be effective for analysis of texture images based on experimental results.

FUNDAMENTAL DEFINITIONS

Before defining adjacency relations, we will give several preliminary definitions concerning digital picture processing here.

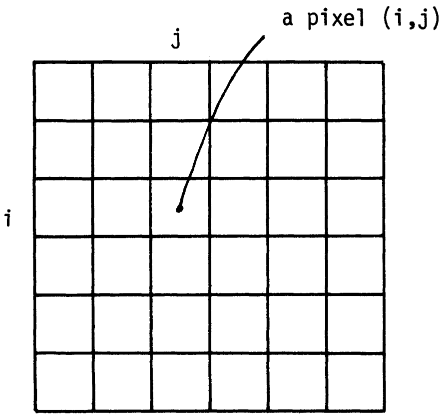


Fig.1 A digitized picture.

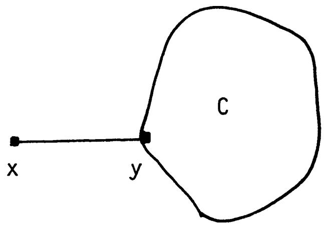


Fig.2 Distance between a pixel x and a set C.

Let (i, j) denote a picture element or a pixel located in the ith row and the jth column, and $\mathbf{F} = \{f_{ij}\}$ denote a digitized picture in which a value of density at a pixel (i, j) is given by f_{ij}. We assume that **F** is a binary picture, that is, f_{ij} is equal to 1 or 0. These pixels are called 1-pixels or 0-pixels respectively (Fig. 1).

The t-neighbor distance (t=4, 8) between two pixels x = (i, j) and y = (k, l) are defined by:

$$d4(x, y) = |i-k| + |j-l| \quad \text{(4-neighbor distance)}$$

$$d8(x, y) = \max(|i-k|, |j-l|) \quad \text{(8-neighbor distance)}$$

The t-neighborhood Nt(x) of a pixel x is defined as the set of pixels y satisfying dt(x, y) = 1.
The distance between a pixel x and a connected component C is defined by (Fig. 2):

$$dt(x, C) = \min \{ dt(x, y);\ y \in C \} \quad (t=4, 8)$$

The distance between two connected components C1 and C2 is defined by:

$$dt(C1, C2) = \min \{ dt(x, y);\ x \in C1,\ y \in C2 \} \quad (t=4, 8)$$

ADJACENCY RELATIONS OF CONNECTED COMPONENTS IN A DIGITAL PICTURE

We present here definitions of neighborhood relations among connected components in a digital picture, based on the concepts on the continuous space.

Let us assume that a picture **F** contains n connected components of 1-pixels. Here a connected component may consist of a single pixel as a special case.

The modified digital Voronoi neighbor (MDVN) is defined based on the modified digital Voronoi diagram (MDVD). The MDVD is defined procedurely as follows.

<u>(Definition 1)</u> Let **F** be an input picture. Covert **F** into **G** according to the following procedure (Fig. 3).

(1) Assign different labels of integers to all different connected components.

(2) Expand in all directions each connected component iteratively. Labels of components are carried with this expansion. If more than one different labels arrive at a pixel at the same time, the pixel is given the label −1.

(3) The procedure stops when all 0−pixels in a picture have been given suitable labels including −1.

The resulting picture gives the desired picture **G**, in which each region filled with a label is called the tile of a connected component having the same label. The modified digital Dirichlet tessellation (MDDT) is defined as the set of all tiles, and the Modified digital Voronoi diagram (MDVD) consists of the set of all pixels locating on borders of tiles and all pixels with the label −1.

The precise definitions of MDDT and MDVD were shown in Mase et al (1981). Examples of the MDDT and the MDVD are shown in Fig. 4

<u>(Definition 2)</u> Two components Ci and Cj are said to be in the modified digital Voronoi neighbor (MDVN) each other if any of the following (i) and (ii) holds:

(i) $\exists x \in Ti$, $\exists y \in Tj$ such that $x \in N8(y)$

(ii) $\exists x \in Ti$, $\exists y \in Tj$, $\exists z \in B$ such that $x \in N8(z)$ and $y \in N8(z)$

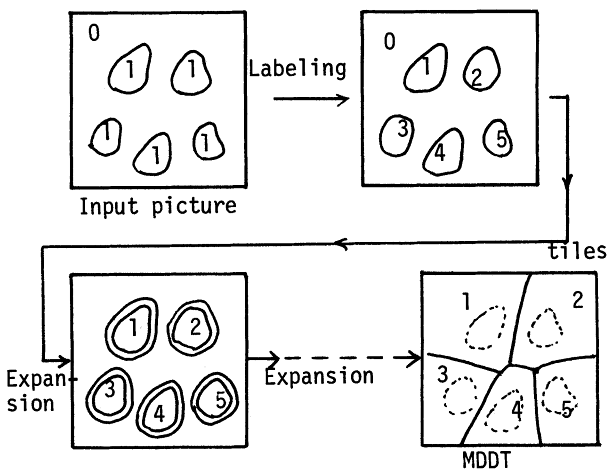
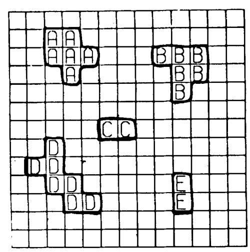
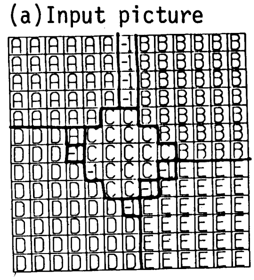


Fig.3 Iterative algorithm to obtain MDDT and MDVD.



(a)Input picture



(b)MDDT

Fig.4 An example of MDDT.

where Ti and Tj are tiles of Ci and Cj , respectively, and B is a set of pixels given the value −1.

The above condition means that tiles of Ci and Cj are adjacent each other in the MDDT.

Next we define the relative neighbor.

(Definition 3) Consider two connected components Ci and Cj in a picture and a pair of pixels x belonging to Ci and y belonging to Cj that are closest each other. For x and y, suppose a rectangular region shared by a square with the center x and the size defined by the distance dt(x, y) and a square with the center y and the same size. This region is called the forbidden region of Ci and Cj. Then, the components Ci and Cj are said to be in the modified relative neighbor each other, if there is no pixel belonging to any component other than the components Ci and Cj in the forbidden region.(Fig. 5).

In the case of continuous space, the relative neighbor was defined by using a common region of circles instead of squares. On a digitized picture plane, squares are used because the 4-neighbor distance or the 8-neighbor distance is employed for convenience of computation.

The modified digital Gabriel neighbor (MDGN) is also defined by extending the Gabriel neighbor by the way similar to the MDRN except for the definition of the forbidden region for two components. Details are omitted here(see Toriwaki et al (1984)).

Two kinds of algorithms were employed to test whether any pixel in other components exist or not in the forbidden region for obtaining the MDRN and the MDGN.

(Algorithm 1) Test whether coordinate values of each pixel is located in the forbidden region or not.

(Algorithm 2) Search the forbidden region on an image plane to find whether any pixel exists in it or not.

Adjacency graph expressions of connected components in a digitized picture can be introduced based on these neighborhood relations. In such graphs, shape features are assigned to nodes and geometrical relations between components to edges as shown in Fig. 6.

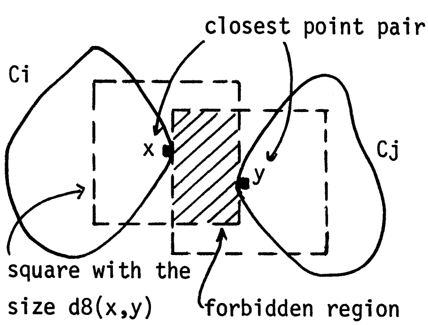


Fig.5 Definition of MDRN。

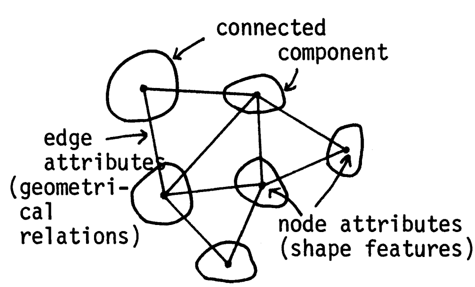


Fig.6 Adjacency graph expression。

Table 1 Computaion time

MDVN	O(L)	
MDRN MDGN	Algorithm1	Algorithm2
	$O(m^3n^3)$	$O(Lm^2n^1)$

n = number of connected components
L = picture size(number of pixels)
m = average number of pixels in a connected component

COMPUTAION TIME AND PROPERTY

Since both the computation time to derive the MDDT and that to obtain the MDVN relations from the MDVN are independent of n (=the number of connected components) (Mase et al:1981), the time to obtain the MDVN t(MDVN) does not depend on the number of components in an input picture. t(MDVN) is approximately proportional to the size of a picture (or the number of pixels).

On the other hand, computation time for MDRN and MDGN depends on which algorithm is employed. In Algorithm 1, the computation time is $O(m^3n^3)$ independent of the picture size where m and n are denoted in Tab. 1. In Algorithm 2, the computation time increases more gradually with n, because the size of a forbidden region decreases on the average as n increases. But it increases with the picture size. The computation time is summarized in Tab. 1.

As a fundamental property, the following relation holds for MDRN and MDGN.

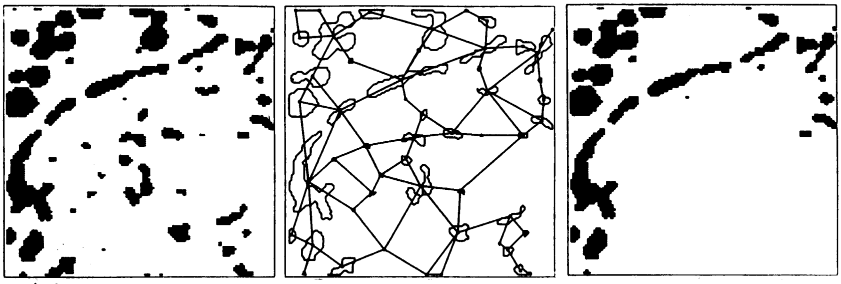
<u>(Property 1)</u> For a set of connected components $\mathcal{C} = \{C1, \ldots, Cn\}$, if Ci and Cj are MDRN, they are also MDGN. In other words, MDGN $\supset$ MDRN.

Inverse of Property 1 is not true in general. There exists no inclusive relation among MDVN and MDGN (or MDRN).

APPLICATIONS TO TEXTURE IMAGE ANALYSIS

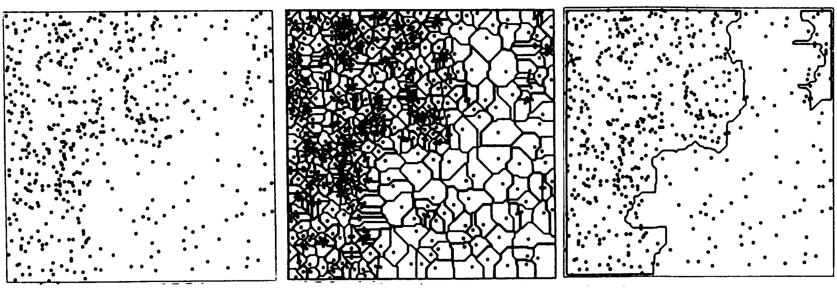
Region tessellation and adjacency graph expression based on neighborhood relations provide tools for analyzing arrangement feature of figures scattered in a digitized picture. Several application examples are presented here.

<u>(Clustering)</u> A set of connected components is clustered by generating the adjacency graphs and removing longer edges. Fig.7 shows a result of extracting only the largest cluster.



(a)Input picture (b)MDRN (c)Extracted cluster

Fig.7 Application to clustering connected components.



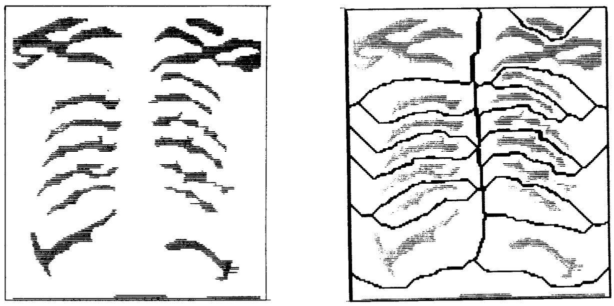
(a)Input picture (b)MDDT (c)Detected edge

Fig.8 Texture edge detection.

(Texture edge detection) Given a connected component set, let us consider to determine a border between the region with the lower density and that with the higher one. After the adjacency graph is obtained by using MDRN, the average distance to the neighboring components is calculated at each node. Tiles corresponding to the components at which the average distance is larger than a suitably selected threshold is eliminated. Then the outer boundary of the set of remaining tiles gives the desired texture edge. Alternatively, the tessellated tile size may be directly thresholded. Fig.8 shows an example, in which the input picture is a point pattern. The result is obtained by leaving smaller edge length in the MDVD. The edge line in the last picture was produced by leaving areas with smaller edge lengths in the adjacency graph expression based on MDVD.

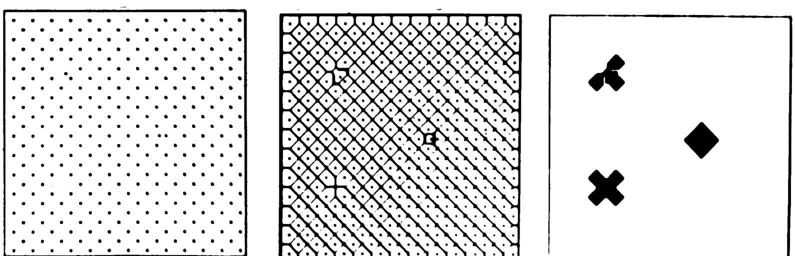
(Region segmentation) In Fig.9, the input picture is a rib image extracted from a chest x-ray image. Fig.9(b) is a result of region segmentation by using the MDVD. In the analysis of chest x-ray images, defining the location of other objects with respect to the ribs is required. The MDDT is used for determination of influence area of each rib image.

<u>(Detection of irregularity of arrangement of points)</u> Fig. 10(a) shows an input picture in which points are arranged regularly. The next picture is given some irregularity by adding and removing several points to(from) the input picture. After constructing the MDVD, the irregular regions are extracted by picking up tiles with different area from the average area of all tiles.



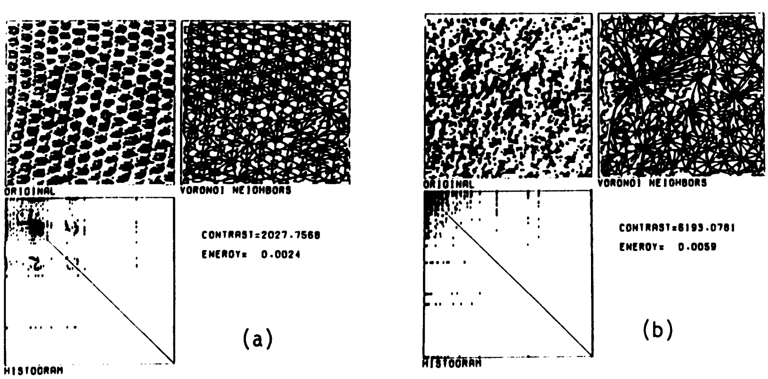
(a)Input picture (b)MDDT

Fig.9 Region segmentation of a chest x-ray image.



(a)Input picture (b) MDDT (c)Detected region

Fig.10 Detection of irregularity in point distribution.



(a) (b)

Fig.11 Texture feature extraction by using cooccurrence matrixes.

(Texture classification by a cooccurrence matrix) A cooccurrence matrix is obtained by counting occurrence of feature value pairs at neighboring nodes. Properties of the cooccurrence matrix are used for texture discrimination. Fig.11 shows the original picture (from Brodatz:1966), the MDVD and the cooccurrence histogram in which each axis corresponds to the size of tiles adjacent in the MDVD. Respective cooccurrence matrixes show characteristic patterns. Images are featured by computing various parameters such as texture energy and contrast as shown in Haralick et al (1973) from these matrixes.

CONCLUSION

In this paper, we presented extensions of the concepts of space tessellations and neighborhood relations in the continuous space to the digitized plane. It was confirmed that they provided new tools for analyzing arrangement features of digital pictures.

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10-5

Q: Can you use some measures of your derived adjacency graphs which would measure the "distance" between two graphs? For example, it seems that you could measure perhaps the coarseness or fineness of some texture patterns by measures of your graphs.

(E. Hall)

A: We now measure similarity of the graphs by an indirect way, that is, by using features of the occurrence matrix derived from the adjacency graph. Some of these features give measures of the coarseness or fineness of textures. Scalor features calculated directry from the adjaceny graph such as the averages of edge lengths and tile areas will also be available. Furthermore it is also possible in principle to compare two graphs directly by using graph matching techniques.