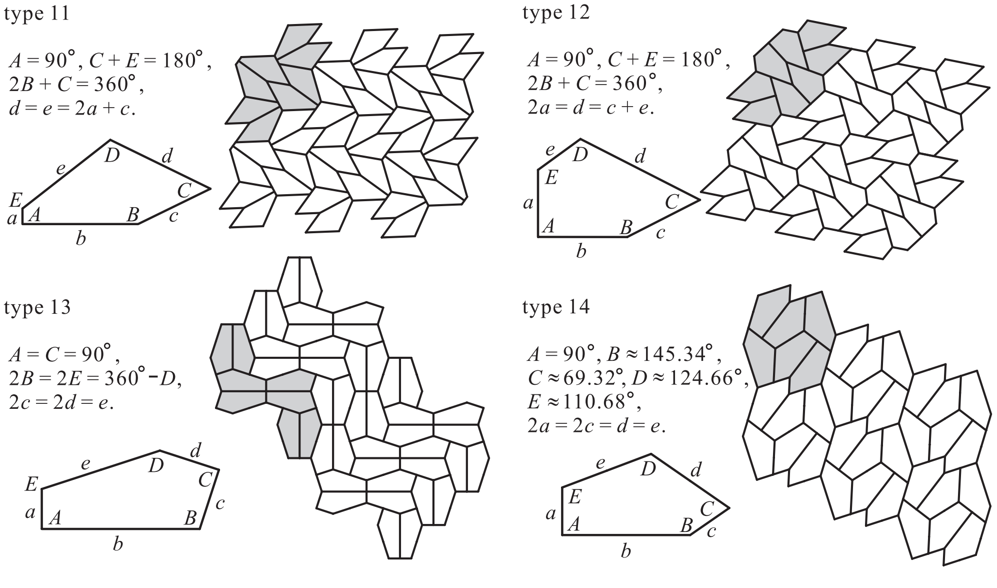

Fig. 2. Convex pentagonal tiles of type 11–14.

(II) Tiling is not necessarily possible with other convex polygons because of constraints on angles and edge lengths. For example, no tiling exists for a polygon if no combination of its interior angles sum to 360°.

(i) In the case of convex hexagons, prototiles can be categorized into three types (REINHARDT, 1918; BOLLOBÁS, 1963; GARDNER, 1975; GRÜNBAUM and SHEPHARD, 1987).

(ii) For convex polygons with seven or more edges, no prototiles exist (REINHARDT, 1918; GRÜNBAUM and SHEPHARD, 1987).

(iii) For the convex pentagonal tiles, there are at present 14 classifications (see Figs. 1 and 2), but it remains unproven whether this is the perfect list of such pentagons (REINHARDT, 1918; KERSHNER, 1968; GARDNER, 1975; SCHATTSCHNEIDER, 1978, 1981; HIRSCHHORN and HUNT, 1985; GRÜNBAUM and SHEPHARD, 1987; WELLS, 1991; SUGIMOTO, 1999; SUGIMOTO and OGAWA, 2000a). Note that, as shown in Figs. 1 and 2, each of convex pentagonal tiles is defined by some conditions between lengths of edges and magnitudes of angles; but some degrees of freedom remain. (However, only the pentagonal tile of type 14 does not have any degrees of freedom except size. For example, the exact value of C in pentagon of type 14 is $\cos^{-1}((3\sqrt{57} - 17)/16)$, and the values of angles B, D, and E can be obtained by C.) Then, unless a convex pentagonal tile is a new prototile, any convex pentagonal tile belongs to one or more of 14 types. The pentagonal case is the only unsolved problem in the study of tiling the plane with congruent convex polygons, and the problem has yet to be approached scientifically (SUGIMOTO and OGAWA, 2003a).

The solution to this sole remaining problem of tiling the plane with convex pentagons requires a systematic approach, that is, the problem should be divided into stages and study