type 11

$A = 90°$, $C + E = 180°$,
$2B + C = 360°$,
$d = e = 2a + c$.

type 12

$A = 90°$, $C + E = 180°$,
$2B + C = 360°$,
$2a = d = c + e$.

type 13

$A = C = 90°$,
$2B = 2E = 360° - D$,
$2c = 2d = e$.

type 14

$A = 90°$, $B \approx 145.34°$,
$C \approx 69.32°$, $D \approx 124.66°$,
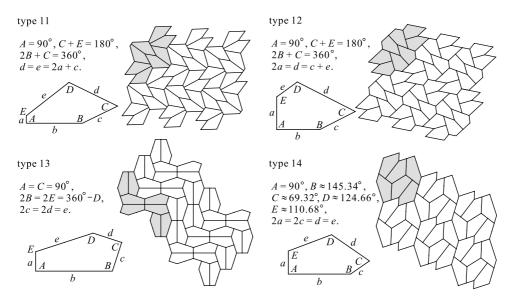$E \approx 110.68°$,
$2a = 2c = d = e$.

Fig. 2. Convex pentagonal tiles of type 11–14.

magnitudes of angles; but some degrees of freedom remain. (However, only the pentagonal tile of type 14 has one degree of freedom, that of size. For example, the exact value of C in pentagon of type 14 is $\cos^{-1}((3\sqrt{57} - 17)/16) \approx 1.2099$ rad $\approx 69.32°$, and the values of angle B, D, and E can be obtained by C.) Then, unless a convex pentagonal tile is a new prototile, any convex pentagonal tile belongs to one or more of 14 types.

In the study of tiling the plane with congruent convex polygons, the case of pentagon is the only unsolved problem. Then we believe that the problem has yet to be approached from a new point of view. For example, the tilings themselves can be distinguished into two kinds by the connecting method: edge-to-edge tilings and non-edge-to-edge tilings. In edge-to-edge tilings, the vertices and edges of polygons coincide with the vertices and edges of the tiling. In non-edge-to-edge tilings, the vertices of polygons may contact the edges of adjoining polygons, that is, there is no restriction on how adjoining polygons meet. As to the pentagonal tilings, though the edge-to-edge tilings are still pentagonal even in topological point of view, the non-edge-to-edge tilings are not. However, the distinction between these two connecting methods is seldom done in the previous studies of convex pentagonal tiling problem. Our interest lies more in edge-to-edge tiling since it is more essential (OGAWA *et al.*, 2001; SUGIMOTO and OGAWA, 2000, 2003a, 2003c, 2005). Therefore, throughout the report, we will consider only edge-to-edge tiling. Hereafter, as long as cautions are unnecessary, an edge-to-edge tiling is written simply a tiling. On the other hand, so far the classification and exhaustive studies of tilings are not very much noticed to the present. However, for convex pentagonal tiles, it will be impossible to express the necessary and sufficient conditions for identifying prototiles without classifying tilings. Therefore, we should pay attention to the properties of tilings. First, in this report,