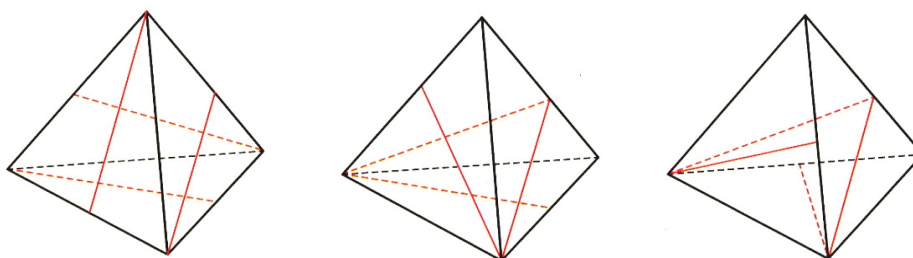


Note that it is left invariant by 4 rotations. Here displacing no points is counted as such a rotation (a rotation of magnitude 0 around any axis), and the other rotations are easily seen to be those of magnitude 180° around the lines connecting the mid-points of the mutually opposite edges. Since there exist altogether 12 rotations which leave the tetrahedron invariant, the above pattern is varied by such rotations in  $12 \div 4 = 3$  ways.

Similarly, each of the patterns, which remain invariant by the 180 degree rotation around a line connecting two opposite edges,



changes in  $12 \div 2 = 6$  ways.

It is easy to see that there exist only these 4 illustrated patterns (and their equivalents obtained by rotations) which are left invariant by some nontrivial rotation.

Therefore, any of the other patterns varies by any of the nontrivial rotations.

As a result we have an equation

$$3 \times 1 + 6 \times 3 + 12 \times \square = 81.$$

Here we have in the box  $\square$  the number of mutually inequivalent patterns each of which varies in 12 ways under rotation.

Since  $3 \times 1 + 6 \times 3 + 12 \times 5 = 81$ , we know that the totality of inequivalent patterns under rotation is  $1 + 3 + 5 = 9$ .

(2) Similarly as in the case of tetrahedron, by counting the patterns at first without considering the effect of rotations, we obtain the number  $2^6 = 64$ .

Since the number of rotations moving the cube to itself is 24, as one can count it by finding the axes and angles of rotations, one can classify the diagonal patterns on the cube