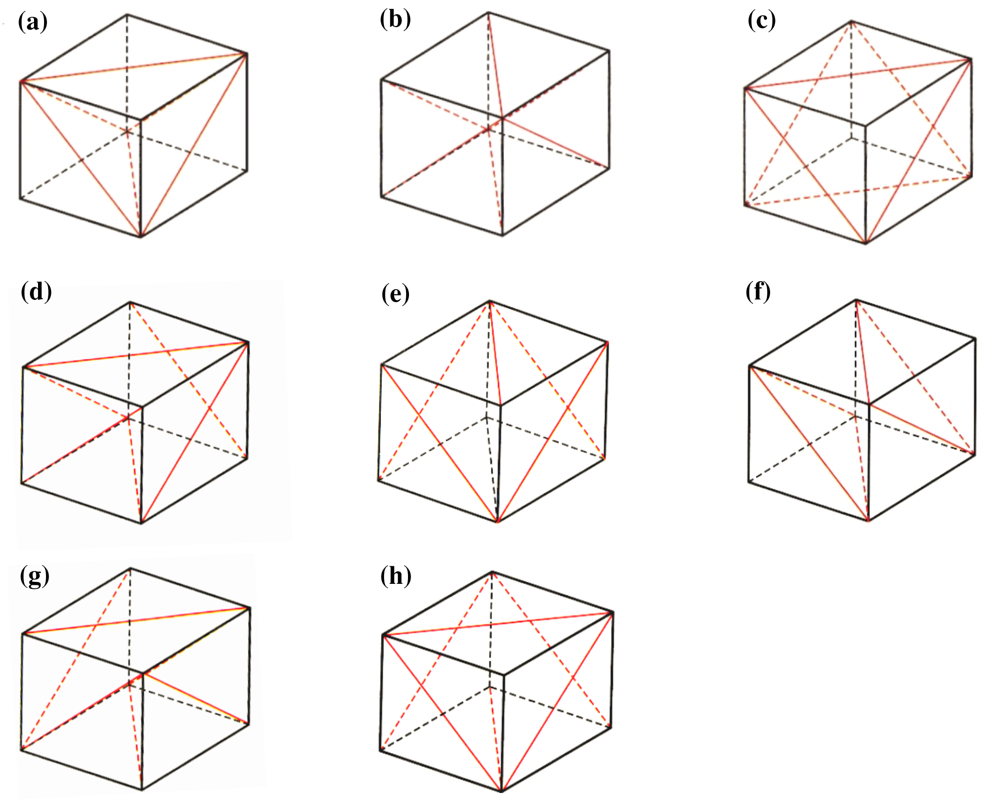

Fig. 1. Diagonal patterns on the cube.

by listing up the patterns according to a priori possible orders 24, 12, 8, 6, 4, 3, 2, 1 of rotational symmetry. Here the order means the number of rotations fixing the pattern.

As a pattern of order 12 one may take (Fig. 1(a)) which varies in $24 \div 12 = 2$ ways. Similarly the two patterns (b) and (c) (Fig. 1) vary in $24 \div 6 = 4$ ways, the three patterns (d), (e) and (f) (Fig. 1) in $24 \div 4 = 6$ ways, the pattern (g) (Fig. 1) in $24 \div 2 = 12$ ways, and the pattern (h) (Fig. 1), being totally asymmetric, varies in 24 ways.

Since

$$2 \times 1 + 4 \times 2 + 6 \times 3 + 12 \times 1 + 24 \times 1 = 64,$$

the above list exhausts all the possibility.

Hence the total number of inequivalent diagonal patterns on the cube is 8.

(3) The three faces which are invariant under the rotation of 90° are arranged in two ways as