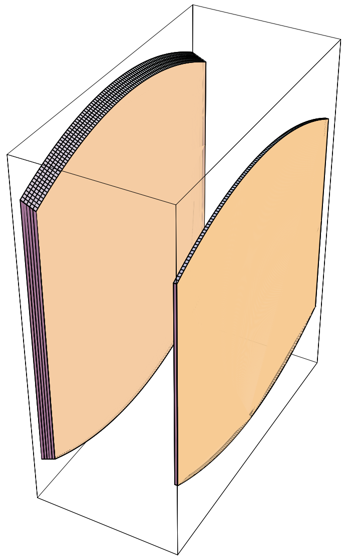

Fig. 13. Samples of Voronoi cells for the case $m = 50$. Five consecutive cells of the points $((m + i)/2m, 0, 0)$, $i = 3, 4, 5, 6, 7$ and one cell of the point $((2m - 1)/2m, 0, 0)$ are shown.

crosses the bottom and top faces of the chosen cube K in points $\mathbf{t}_0 = ((i+1/2)/2m, (j+1/2)/2m, 0)$ and $\mathbf{t}_1 = ((i+1/2)/2m, (j+1/2)/2m, 1)$, respectively. Consider function $f(z) := |\mathbf{t}, \mathbf{x}_i|^2 - |\mathbf{t}, \mathbf{y}_j|^2$, where $\mathbf{t} = ((i+1/2)/2m, (j+1/2)/2m, z) \in l$. It is equal to

$$f(z) = 2z - 1 + \frac{j^2 + 2j - i^2 - 2i}{4m^2}.$$

It is easy to check that $f(0) < 0$ and $f(1) > 0$. Therefore, $f(z_{ij}) = 0$ in some intermediate point z_{ij}: $0 < z_{ij} < 1$. This point corresponds to the circumcenter O_{ij} of a simplex in (13). Thus the circumcenter has to be located inside segment [0, 1] of line l.

Thus, the circumball has no points of X except for the four points mentioned in (13). In fact, in an open layer $0 < z < 1$ there are no points of X at all because the z-coordinate of any point of X is integer. Now plane $z = 1$ crosses the circumball along a circle centered at point $((i+1/2)/2m, (j+1)/2/m, 1)$ and coming through two points from X: $(0, j/2m, 1)$ and $(0, (j + 1)/2m, 1)$. Since $0 \le i \le m - 1$ this circle contains no other points of X in plane $z = 1$. The analogous fact is true for plane $z = 0$. Points of X which are placed on planes $z = k$, where integer $k \ne 0, 1$, do not enter the circumball at all, because the circumcenter O_{ij} lies between planes $z = 0$ and $z = 1$. This geometrical fact implies that any point $\mathbf{x} = (l/2m, n, k) \in \Lambda_0$ (l, n are integer and k is even) if $k \ne 0$ is situated further away O_{ij} than point $\mathbf{x}_i$. Respectively, a point from Λ_1, if it does not lie in plane $z = 1$, is further away O_{ij} than $\mathbf{y}_j$. Inspect this fact for point $\mathbf{x} \in \Lambda_0$. Since $k \ne 0, 1$ we have