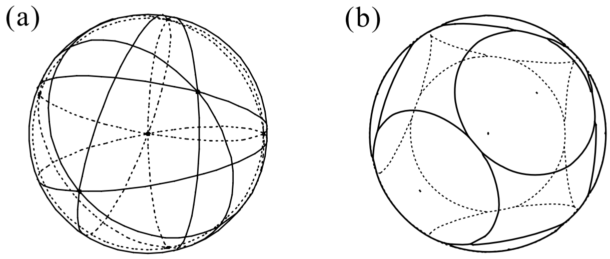

Fig. 6. (a) Our sequential covering for $N = 7$. (b) Our solution of Tammes problem for $N = 7$. Both viewpoints are (0, 0, 10). In this example, the coordinates of the centers are respectively (0, 0, –1), (0.16977, –0.96282, –0.21014), (0.97767, 0, –0.21014), (0.16977, 0.96282, –0.21014), (–0.91871, 0.33438, –0.21014), (–0.55588, –0.46644, 0.68806), and (0.39850, 0.33438, 0.85404).

can cover the region except for a point in the kite under the Minkowski condition. Therefore, we find that $\bar{r}_6$ is the largest spherical distance in the kite. It is obvious that the kite $K_8K_4K_6K_7$ on the sphere is inside the spherical quadrilateral $K_8K_4K_6K_7$ for fixed vertices K_8, K_4, K_6, and K_7 (see Fig. 5). From (8) and the coordinates of K_8, K_4, K_6, and K_7 in Appendix A.1, we find numerically that four vertices of the spherical quadrilateral $K_8K_4K_6K_7$ satisfies the relations $d_s(K_6, K_8) > d_s(K_4, K_7)$, and $\pi/2 \geq d_s(K_6, H) = d_s(H, K_8) = d_s(K_6, K_8)/2 > d_s(H, K_4) = d_s(H, K_7) > 0$ for $\tan^{-1}2 \leq r < \pi/2$. Note that H is the middle point of the geodesic arc K_6K_8. Therefore, from Corollary of Theorem 2, the farthest pair of points in the spherical quadrilateral $K_8K_4K_6K_7$ is the pair of K_6 and K_8; namely $d_s(K_6, K_8)$ is the largest spherical distance in the kite $K_8K_4K_6K_7$ and is equal to $\bar{r}_6$. Then, from (9) and the coordinates of K_6 and K_8 in Appendix A.1, we have

$$r = d_s(K_6, K_8) = \cos^{-1}\left(\frac{41\cos^4 r - 8\cos^3 r - 18\cos^2 r + 1}{9\cos^4 r + 8\cos^3 r - 2\cos^2 r + 1}\right). \tag{10}$$

In addition, we note $K_8 \in \partial C_5$. Thus, $\cup_{v=1}^{6} C_v$ is in an extreme state (the set $\cup_{v=1}^{6} C_v$ covers S except for the point K_6) if and only if M_6 is put on the point K_8. At this time, K_6 is the unique uncovered point. Equation (10) is solved against r by using mathematical software. As a result, the value of the upper bound for $N = 7$ is obtained

$$r_7 = \bar{r}_6 = \cos^{-1}\left(1 - \frac{4}{\sqrt{3}}\cos\left(\frac{7\pi}{18}\right)\right) \approx 1.35908 \quad \text{rad.} \tag{11}$$

Therefore, when M_6 and M_7 are put at K_8 and K_6, respectively, then $\cup_{v=1}^{7} C_v$ which contains W_6 covers the whole of S (see Fig. 6(a)). Namely, our sequential covering for $N = 7$ is completed.

Here, we check whether the position of points K_2 and K_5 for M_4 and M_5 satisfy the condition that $\cup_{v=1}^{4} C_v$ and $\cup_{v=1}^{5} C_v$ are in an extreme state, respectively. For that purpose,