Solving Infinite Kolam in Knot Theory

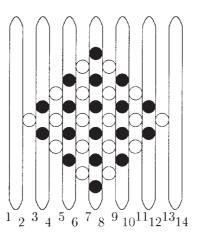


Fig. 4. Braid representatives with cups and caps inserted, or Morse link presentations of the family of the Kolam diagrams. For convenience, numbers at the bottom are assigned to the vertical lines.

Accordingly, the total number of Kolam patterns for a given grid pattern of dots is given by

$$2^{D}$$
, (2)

where *D* denotes the number of the black and white sites:

$$D = \begin{cases} 4 & \text{for } (1 - 3 - 1) \\ 16 & \text{for } (1 - 3 - 5 - 3 - 1) \\ 36 & \text{for } (1 - 3 - 5 - 7 - 5 - 3 - 1) \\ 4N^2 & \text{for } (1 - 3 - \dots - (2N + 1) - \dots - 3 - 1), \ N = 1, 2, 3, \dots \end{cases}$$
(3)

This obviously shows that the number of cases increases astronomically as the size of the diamond Kolam grows.

Following Rules 2.2, there is left no choice for the lines except at the black and white sites. Namely, the lines except at the B/W sites are the ones connecting the neighbourhood B/W sites and the ones encircling boundary dots. Consequently, the dots are found to be irrelevant after this translation so that they can be removed from the patterns, leaving only diagrams of the lines. Hence, the problem can be reduced to that of the knot group acting on the given vertical lines (14 lines for the above case) with caps and cups on the boundary dots (Fig. 4). In the next subsection, we describe the diagrams and analyze them in knot theory. Note that this translation can be shown equivalent to the mirror curves (GERDES, 1990; JABLAN, to appear) for this particular grid pattern.