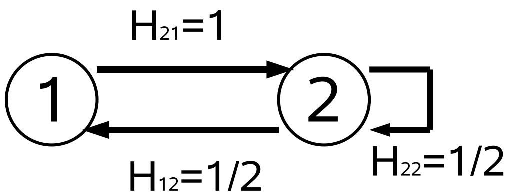

Fig. 1. Directed network consisting of two nodes in which node 1 is always directed to node 2, and node 2 to both with equal probability.

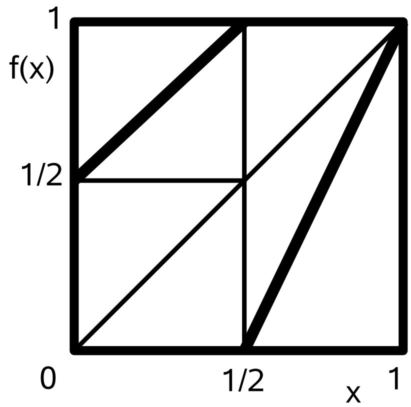

Fig. 2. Piecewise-linear map corresponding to the directed network consisting of two nodes in which node 1 is always directed to node 2, and node 2 to both with equal probability.

map satisfying $f(I_1) = I_2$, $f(I_2) = I_1 \cup I_2$ is piecewise linear, and is given by $f(x) = x + 1/2$ $(0 \le x < 1/2)$, $2x - 1$ $(1/2 \le x \le 1)$ with $I_1 = [0,1/2)$ and $I_2 = [1/2,1)$ as shown in Fig. 2. This is a simple example of Markov partition. The slope $f'(I_1) = 1$ is equal to the output degree of node 1, $f'(I_2) = 2$ to that of node 2. Note that the slope of the map is equal to the output degree of the graph in this way and that the element of H is equal to the inverse of a slope or of an output degree. This map is a special case of Kalman's map, whose static and dynamical properties reproduce those of a discrete-time discrete-state Markov process characterized by the aforementioned transition matrix H (KALMAN, 1956; KOHDA and FUJISAKI, 1999).

In the case of chaotic dynamics caused by a one-dimensional map f, the trajectory is given by iteration. Its distribution at time n, $\rho_n(x)$, is given by the average of this delta function $\langle \delta(x_n - x) \rangle$, where $\langle \cdots \rangle$ denotes the average with respect to initial points x_0. The temporal evolution of ρ is given by the following relation

$$\rho_{n+1}(x) = \int_0^1 \delta\big(f(y) - x\big)\rho_n(y)dy \equiv \mathcal{H}\rho_n(x).$$