(a) (b)

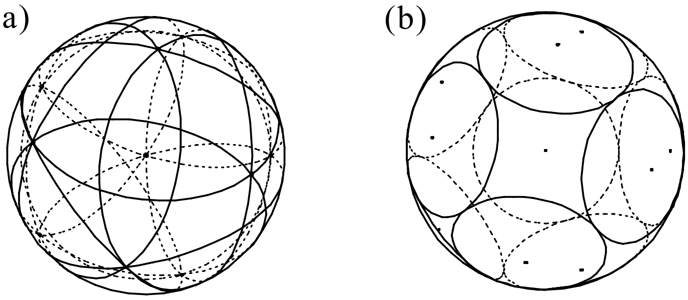

Fig. 5. (a) Our sequential covering for $N = 10$. (b) Our solution of Tammes problem for $N = 10$. Both viewpoints are (0, 0, 10). In this example, the coordinates of the centers are respectively (0, 0, –1), (0.26335, –0.87585, –0.40439), (0.91458,0, –0.40439), (0.26335, 0.87585, –0.40439), (–0.76292, 0.50440, –0.40439), (–0.77575, –0.57681, –0.25593), (–0.13883, –0.78326, 0.60599), (–0.79006, 0.092588, 0.60599), (0.084546, 0.74290, 0.66405), and (0.735778, –0.13295, 0.66405).

quadrangle on S. Then, from Corollary of Theorem 2 in I and the relations for four vertices of $(W_8)^c$, we find that $d_s(K_{11}, K_{12})$ is equal to the spherical distance of the largest interval in the uncovered region $(W_8)^c$. Therefore, $\bar{r}_9$ is equal to (10). In addition, we find that $\cup_{v=1}^{9} C_v$ is in an extreme state if and only if M_9 is put on the point $K_{12} \in \partial C_8$. Therefore, we choose M_9 on K_{12}, and then K_{11} is the unique uncovered point on S.

For $N = 10$, we initially assumed that r should be in the range $(\tan^{-1}2, \cos^{-1}(1/3)]$. Then, as a result of the investigation, our upper bound r_{10}, (10), has fallen within the range $(\tan^{-1}2, r_9]$. However, one might suspect that the fact is due to the assumption. So, if r is in the range $(0, \tan^{-1}2]$, we examine whether W_9 is able to cover S except for finite points. When r is assumed to be equal to $\tan^{-1}2$, we find that the set W_9 leave an uncovered region on S. For its detail, refer to the consideration of Subsec. 3.2. Furthermore, for $0 < r < \tan^{-1}2$, the uncovered region would become still bigger. Hence, the upper bounds r_{10} cannot be in the range $(0, \tan^{-1}2]$ as in the cases of $N = 8$ and 9 in I. Thus, we note that our initial assumption $\tan^{-1}2 < r \le r_9$ is also confirmed ($r = \tan^{-1}2$ is excluded from the above consideration).

As a result of consideration above, the set W_9 covers S except for K_{11}. Therefore, when M_{10} is put at the point K_{11}, the whole of S is covered by $\cup_{v=1}^{10} C_v$ which contains W_9 (see Fig. 5(a)). Thus, our consideration that our r is equal to the upper bound r_{10} (a side-length of the spherical square $K_9K_{11}K_{12}K_{10}$ which satisfies (9)) is confirmed and (10) is certainly the upper bound for $N = 10$.

3.2. N = 11 and 12

It is expected that the solution r_{11} for $N = 11$ should not be larger than r_{10}. Then, we assume $r_{11} \le r \le r_{10}$. Further, we assume $\tan^{-1}2 \le r_{11}$. Hence, the relations (7) and (8) hold because of the assumption $\tan^{-1}2 \le r_{11} \le r \le r_{10}$. Then, we can use the same configuration of the first five spherical caps of the case $N = 10$. When the fifth spherical cap C_5 is put on the sphere, in the same way as the foregoing subsection, a quadrilateral kite $K_8K_4K_6K_7$ on the sphere might be formed as the uncovered region. Further, because of the condition that