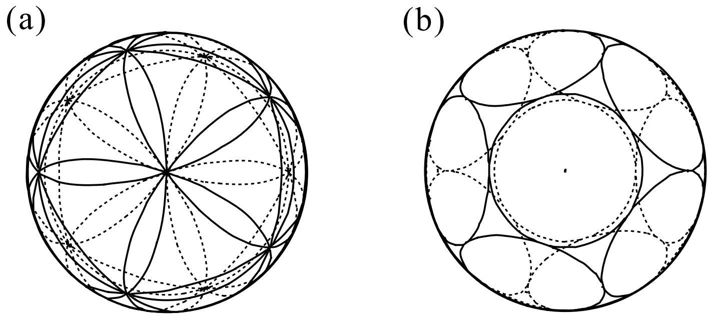

Fig. 8. (a) Our sequential covering for $N = 12$. (b) Our solution of Tammes problem for $N = 12$. Both viewpoints are (0, 0, 10). In this example, the coordinates of the centers are respectively (0, 0, –1), (0.27639, –0.85065, –0.44721), (0.89443,0, –0.44721), (0.27639, 0.85065, –0.44721), (–0.72361, 0.52573, –0.44721), (–0.72361, –0.52573, –0.44721), (–0.27639, –0.85065, 0.44721), (–0.89443,0, 0.44721), (–0.27639, 0.85065, 0.44721), (0.72361, 0.52573, 0.44721), (0.72361, –0.52573, 0.44721), and (0, 0, 1).

$\cup_{\nu=1}^{10} C_\nu$ is in an extreme state automatically and that S is covered by the set W_{10} except for two points K_4 (or K_{11}) and P (the center point of spherical regular pentagon $K_9K_4K_6K_7K_{10}$) from the relation (14). At this time, we see that P is the cross point of perimeters ∂C_7, ∂C_8, and ∂C_9. Furthermore, as a result of calculation by using the coordinates of K_9, K_{10}, and K_7 for $r = \tan^{-1}2$, P is just the north pole (0, 0, 1) certainly. Therefore, if M_{11} is put on $K_4 \in \partial C_{10}$, $\cup_{\nu=1}^{11} C_\nu$ is in an extreme state and P is a unique uncovered point on S. Then, in order to cover whole of S, we have to put one more cap at P. In other words, when $r = \tan^{-1}2$, according to our sequential covering, we can put twelve caps on S under Minkowski covering. Therefore, M_{11} and M_{12} are put on K_4 and P, respectively, then $\cup_{\nu=1}^{12} C_\nu$ which contains W_{11} covers the whole of S. Namely, we are able to see that $d_s(P, K_4) = \tan^{-1}2 = \bar{r}_{11}$. Thus, we consider that $\tan^{-1}2$ is the upper bound r_{12} for $N = 12$. As a result, we find that the positions of caps of our sequential covering for $N = 12$ correspond to the regular icosahedral vertices (see Fig. 8(a)). Therefore, if all spherical caps of our sequential covering for $N = 12$ are replaced by half-caps, all of those half-caps contact with other five half-caps and there is no space for those half-caps to move.

Then, how about r_{11}? From the results of $N = 10$ and 12, it becomes obvious that our initial assumption $r_{12} = \tan^{-1}2 \le r \le r_{10}$ is indispensable. At the time the center M_6 is placed on the point K_8, we had to place five more caps of the angular radius r on the uncovered region $(W_6)^c$ under the Minkowski covering. Then, from the above considerations, we see that the maximal spherical regular pentagon of side-length r ($r_{12} \le r \le r_{10}$) on $(W_6)^c$ is identical with the spherical regular pentagon $K_9K_4K_6K_7K_{10}$ which satisfies the relation (14). So, we conclude that r_{11} is equal to $r_{12} = \tan^{-1}2$ and use the same configuration of the first eleven spherical caps of the case $N = 12$. However, when $r_{11} = \tan^{-1}2$, our sequential covering for $N = 11$ covers S except for the point P. But, if only the eleventh cap C_{11} is replaced by a closed cap, our eleven caps can completely cover the whole of S under the Minkowski condition. Thus, for $N = 11$, we need a special care as above.