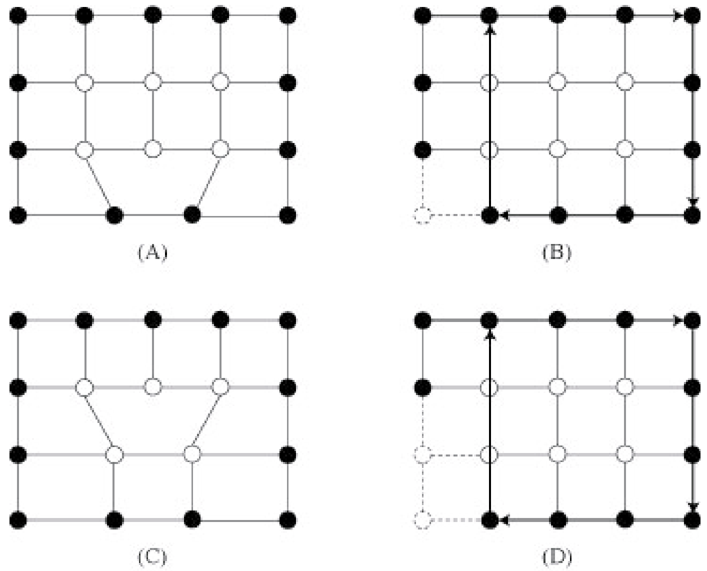

Fig. 3. (A) Lattices with a dislocation whose "strength" is one. (B) Corresponding perfect lattices. (C) Lattices with a dislocation whose "strength" is two. (D) Corresponding perfect lattices.

$$\chi = \sum_{n=0}^{1} (-1)^n \alpha_n = \alpha_0 - \alpha_1, \tag{3}$$

where α_n is the number of n-simplex. Since we regard crystal lattice as one-dimensional complex made of 0- and 1-simplices, so we ignored the higher order terms ($n \geq 2$) of Eq. (3). From Eqs. (2) and (3), the first Betti number is given by

$$b_1 = b_0 - \alpha_0 + \alpha_1. \tag{4}$$

For instance, the first Betti number of lattices in Figs. 1 and 2 are $b_1^{\mathrm{ED}} = 1 - 14 + 20 = 7$, $b_1^{\mathrm{ED(PL)}} = 8$, $b_1^{\mathrm{SD}} = 9$ and $b_1^{\mathrm{SD(PL)}} = 10$ (the meaning of the abbreviation will be mentioned in the next section.). This means that the first Betti number is useful for classifying the four lattices that cannot be distinguished by the zero Betti number ($b_0 = 1$). In the following sections, we will consider the physical meanings of the first Betti number in the crystal lattice as a one-dimensional complex.

3. Dislocations and the First Betti Number

First, to clarify the relationship between dislocations and the first Betti number b_1, we take up the simple case in which the lattices include the edge and the screw dislocations have unit length (Figs. 1(A) and 2(A), see also Fig. 2(C)). To introduce Burgers vector, we