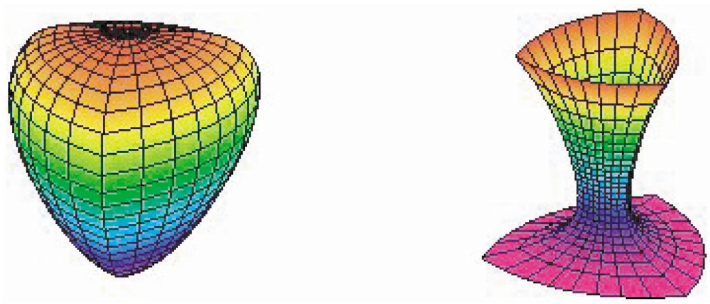
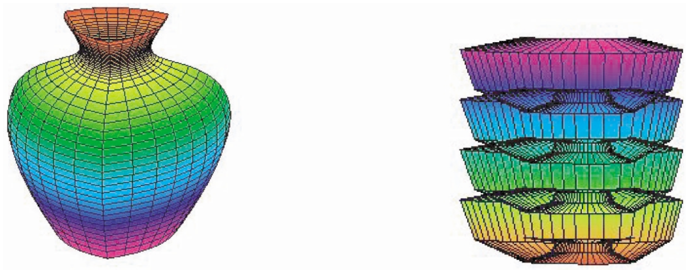


(a)



(b)

Fig. 8. (a) left: $(m, n_1, n_2, n_3, M, N_1, N_2, N_3) = (3, 3.2, 4, 4, 3, 10/9, 10/9, 10/9)$, Wulff shape. right: a generalized anisotropic catenoid. (b) left: $(m, n_1, n_2, n_3, M, N_1, N_2, N_3, \Lambda, c) = (3, 3.2, 4, 4, 3, 10/9, 10/9, 10/9, 0.7, 1)$, a generalized anisotropic unduloid for the Wulff shape in (a). right: $(m, n_1, n_2, n_3, M, N_1, N_2, N_3, \Lambda, c) = (3, 3.2, 4, 4, 3, 10/9, 10/9, 10/9, 1, -2)$, a generalized anisotropic nodoid for the Wulff shape in (a).

where $(\alpha(t), \beta(t))$ is a convex curve, and $\Gamma_W : (u(\sigma), v(\sigma))$ is a convex curve which is symmetric with respect to the v-axis. We assume here that σ is arc length parameter of $(u(\sigma), v(\sigma))$. We remark that all the curves obtained by intersecting W with horizontal planes are homothetic (similar) to the curve $(\alpha(t), \beta(t))$, so they are homothetic to each other.

Let Σ be a smooth surface such that all the curves obtained by intersecting Σ with horizontal planes are homothetic to the curve $(\alpha(t), \beta(t))$. Then Σ is given by

$$X(s, t) = (x(s)\alpha(t), x(s)\beta(t), z(s)), x(s) \geq 0, \qquad (10)$$

using a smooth curve $\Gamma_\Sigma : (x(s), z(s))$ with arc length s.

The anisotropic Gauss map $\omega : \Sigma \to W$ can be regarded as a mapping which maps a point $(x\alpha(t), x\beta(t), z)$ in Σ to a point $(u\alpha(t), u\beta(t), v)$ in W. This means that u can be regarded locally as a function of x through ω.

Recall that, at each point p in Σ, $\Lambda(p)$ is the sum of the stretch rates of the anisotropic Gauss map $\omega : \Sigma \to W$ for any two orthogonal directions. In the present case, we can take the vertical direction and the horizontal direction as these two directions. Recall that σ, s are arc lengths of the profile curves Γ_W, Γ_Σ, respectively. Because of this, the stretch rate of ω for the vertical direction is $d\sigma/ds$, which is the rate of σ with respect to s. On the other hand, it is clear that the stretch rate of ω for the horizontal direction is u/x. Therefore, Σ is a surface with constant anisotropic mean curvature if and only if

$$\Lambda = d\sigma/ds + u/x \equiv \text{constant} \qquad (11)$$

on the whole surface. Because of the definition of ω, the tangent to Γ_Σ at a point $(x(s), z(s))$ coincides with the tangent to Γ_W at the corresponding point $(u(\sigma), v(\sigma))$. This means that $dx(s)/ds = du(\sigma)/d\sigma$, $dz(s)/ds = dv(\sigma)/d\sigma$ holds. Therefore,

$$du/dx = d\sigma/ds = dv/dz \qquad (12)$$

holds. Hence, Eq. (11) can be written as

$$du/dx + u/x = \Lambda \equiv \text{constant}, \qquad (13)$$

which is equivalent to

$$x(du/dx) + u = \Lambda x. \qquad (14)$$

Integrating (14) with respect to x, we obtain

$$ux = \Lambda(x^2/2) + c/2, \qquad (15)$$

where c is any constant. In the case where $\Lambda \neq 0$, we obtain

$$x = (1/\Lambda)\left(u \pm \sqrt{u^2 - \Lambda c}\right), \qquad (16)$$

while if $\Lambda = 0$, we obtain

$$x = c/(2u). \qquad (17)$$