

Fig. 2. A circular city with radial-circular network and the shortest routes between two points.

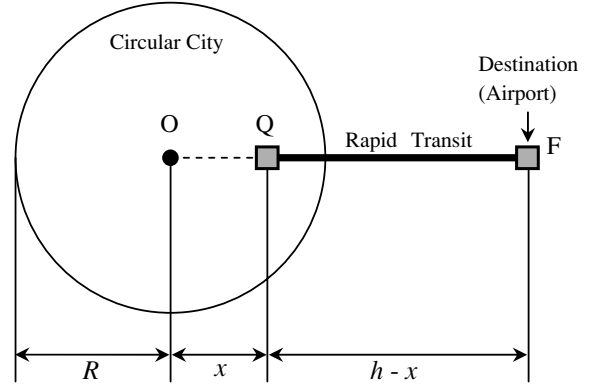


Fig. 3. A simple geometric model of rapid transit system.

(i) $0 \leq x \leq R/3$

$$\psi(s|x) = \begin{cases} \frac{s^2 + 4xs}{\pi R^2 x} & (0 \leq s \leq x) \\ \frac{1}{\pi R^2 x} \{s^2 + (2\pi - 4)xs + (8 - 2\pi)x^2\} & (x < s \leq 2x) \\ \frac{1}{\pi R^2} \{2\pi s - (2\pi - 4)x\} & (2x < s \leq R - x) \\ \frac{1}{\pi R^2 x} \{-s^2 + (2\pi - 2)xs - (2\pi - 3)x^2 + R^2\} & (R - x < s \leq R + x) \end{cases}$$

(ii) $R/3 < x \leq R/2$

$$\psi(s|x) = \begin{cases} \frac{s^2 + 4xs}{\pi R^2 x} & (0 \leq s \leq x) \\ \frac{1}{\pi R^2 x} \{s^2 + (2\pi - 4)xs + (8 - 2\pi)x^2\} & (x < s \leq R - x) \\ \frac{1}{\pi R^2 x} \{(2\pi - 6)xs + (7 - 2\pi)x^2 + R^2\} & (R - x < s \leq 2x) \\ \frac{1}{\pi R^2 x} \{-s^2 + (2\pi - 2)xs - (2\pi - 3)x^2 + R^2\} & (2x < s \leq R + x) \end{cases}$$

(iii) $R/2 < x \leq R$

$$\psi(s|x) = \begin{cases} \frac{s^2 + 4xs}{\pi R^2 x} & (0 \leq s \leq R - x) \\ \frac{1}{\pi R^2 x} \{2xs - x^2 + R^2\} & (R - x < s \leq x) \\ \frac{1}{\pi R^2 x} \{(2\pi - 6)xs + (7 - 2\pi)x^2 + R^2\} & (x < s \leq 2x) \\ \frac{1}{\pi R^2 x} \{-s^2 + (2\pi - 2)xs - (2\pi - 3)x^2 + R^2\} & (2x < s \leq R + x) \end{cases}$$

(7)

The average value of the distance s between homes distributed over a circular city to the fixed terminal station is obtained by the following integration:

$$E(s|x) = \int_0^{R+x} s \psi(s|x) ds. \quad (10)$$

By calculating Eq. (10) using Eqs. (7), (8) and (9), we obtain the same value of $E(s|x)$ which is the cubic function of the location of the terminal station x :

$$E(s|x) = \frac{2}{3\pi R^2} x^3 + \left(1 - \frac{2}{\pi}\right)x + \frac{2R}{3}. \quad (11)$$

3. Formulation of Problems

(8) 3.1 Minimization of the average access time

Let us denote the average access time by $f(x)$ as a function of the location of the terminal station x . The average access time from uniformly distributed points in the city to the airport is the combination of the average access time to the terminal station and the time to the airport by using the rapid transit. Therefore, $f(x)$ is given as follows:

$$f(x) = \frac{1}{w} \left\{ \frac{2}{3\pi R^2} x^3 + \left(1 - \frac{2}{\pi}\right)x + \frac{2R}{3} \right\} + \frac{h-x}{v}. \quad (12)$$

Our aim here is to find $x = x^*$ which minimizes the average access time to the airport:

$$\min_x f(x) \quad (13)$$

(9)

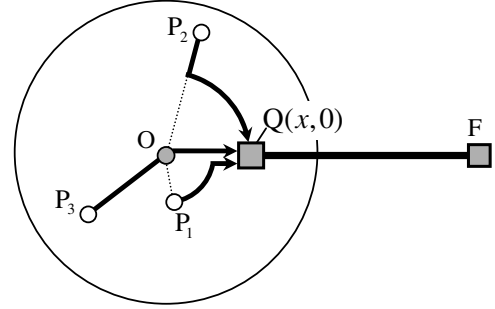


Fig. 4. Access to the airport by three different origins over a radial-circular network.