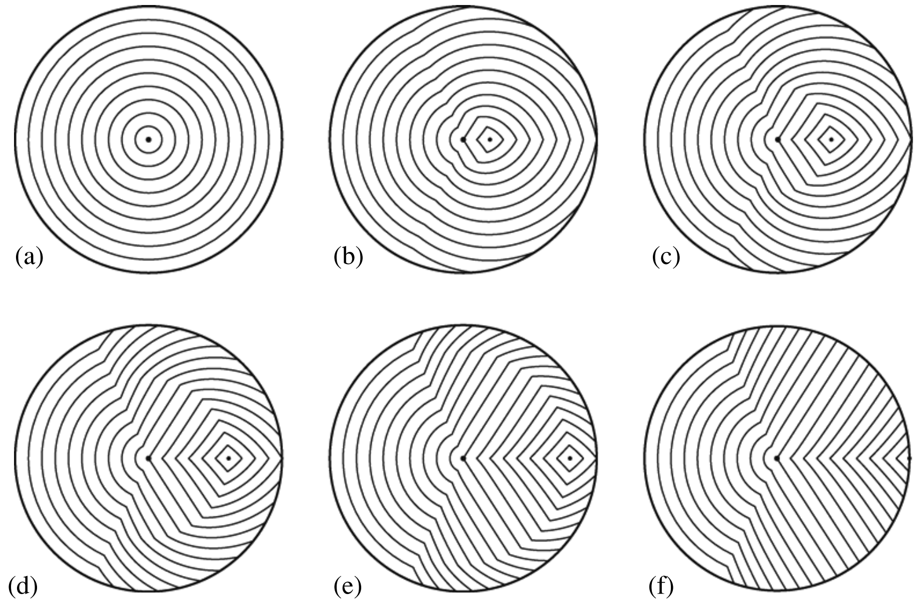


Fig. 5. Equi-distance contours to various locations of fixed points Q($x$, 0): (a) $x = 0$; (b) $x = 0.2R$; (c) $x = 0.4R$; (d) $x = 0.6R$; (e) $x = 0.8R$; (f) $x = R$.

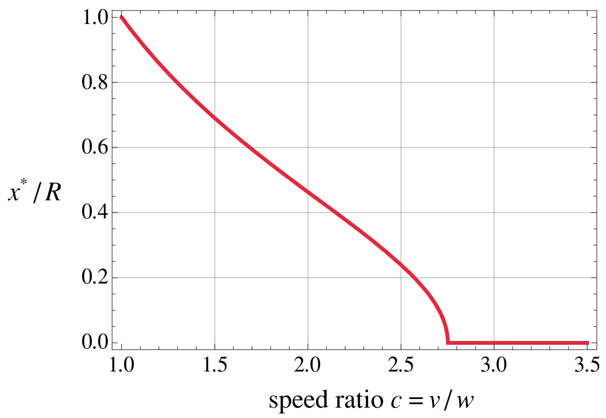


Fig. 6. The optimal location of the terminal station as a function of the speed ratio of intra-city transportation and rapid transit system.

The first and second derivatives of $f(x)$ is given as follows:

$$f'(x) = \frac{1}{w}\left(\frac{2}{\pi R^2}x^2 + 1 - \frac{2}{\pi}\right) - \frac{1}{v}, \tag{14}$$

$$f''(x) = \frac{1}{w} \cdot \frac{4x}{\pi R^2}. \tag{15}$$

These equations show $f(x)$ is a strictly convex function.

Let $x = \tilde{x}$ be the solution of the equation of $f'(x) = 0$. Then $\tilde{x}$ is obtained as follows:

$$\tilde{x} = R\sqrt{\frac{\pi}{2}\left(\frac{w}{v} - \frac{\pi - 2}{\pi}\right)}. \tag{16}$$

The value of the first derivative at the peripheral of the city, $x = R$, is given by

$$f'(R) = \frac{1}{w} - \frac{1}{v}. \tag{17}$$

This indicates $f'(R) > 0$ by the assumption of $w < v$.

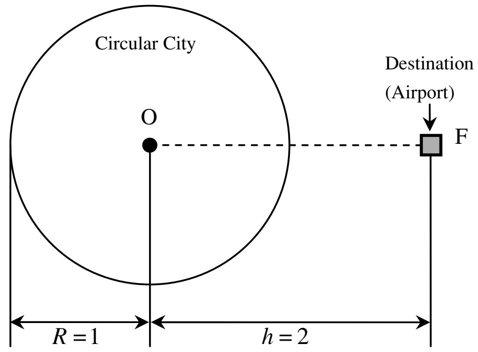


Fig. 7. The parameter setting for the following numerical example.

Consequently, the optimal location of the station is given by $x^* = \tilde{x}$ when $0 \leq \tilde{x}$ and $x^* = 0$ otherwise. From the above argument, the minimizer of $f(x)$ is given as follows:

$$x^* = \begin{cases} 0 & \text{when } \dfrac{v}{w} \geq \dfrac{\pi}{\pi - 2}, \\ R\sqrt{\dfrac{\pi}{2}\left(\dfrac{w}{v} - \dfrac{\pi - 2}{\pi}\right)} & \text{when } \dfrac{v}{w} < \dfrac{\pi}{\pi - 2}. \end{cases} \tag{18}$$

Equation (18) shows that if the rapid transit system has sufficiently high rapid, that is the speed ratio $c = v/w$ is more than $\pi/(\pi - 2) \approx 2.752$, the desirable location of the terminal station is at the city center. Figure 6 shows the optimal location $x^*/R$ as a function of the speed ratio $c = v/w$.

### 3.2 Maximization of the number of users accessible to the airport within a given time

The number of people accessible to a fixed point (such as an important facility) within a given time is an important measure for evaluating the accessibility of the point under study. In this section, we consider the problem of maximiz-