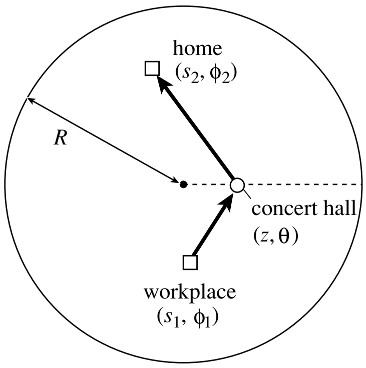

Fig. 1. A circular city of radius R.

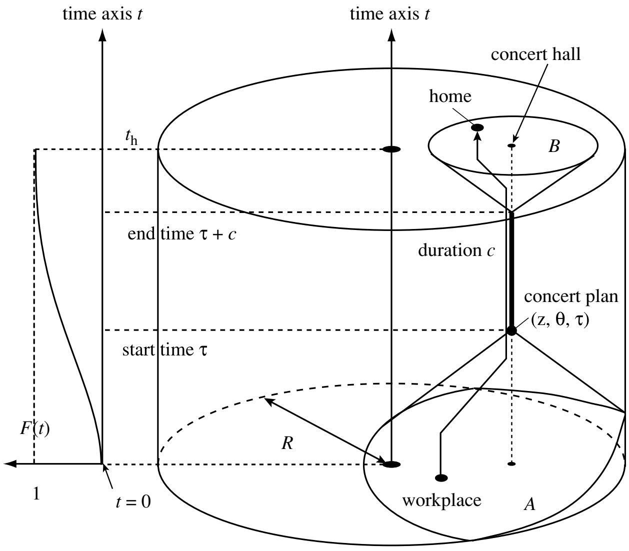

Fig. 2. Movement of a potential customer in the space-time region.

(ii) The cumulative distribution of the departure time t of commuters is given by $F(t)$;
(iii) The distance between any two points is measured by Euclidean distance;
(iv) The traveling speed of commuters is given by a fixed constant v.

The function $\rho(s_1, \phi_1, s_2, \phi_2)$ is called the trip density which is the continuous counterpart of the origin-destination (OD) matrix (Vaughan, 1987). By the definition of trip density, the number of commuters having workplaces within a small region at (s_1, ϕ_1) with area $s_1 \mathrm{d}s_1 \mathrm{d}\phi_1$ and having homes within a small region at (s_2, ϕ_2) with area $s_2 \mathrm{d}s_2 \mathrm{d}\phi_2$ is given by

$$\rho(s_1, \phi_1, s_2, \phi_2) s_1 s_2 \mathrm{d}s_1 \mathrm{d}\phi_1 \mathrm{d}s_2 \mathrm{d}\phi_2.$$

The departure time distribution of commuters means the proportion of commuters that can leave their workplace by time t. In this paper, we assume that $F(t)$ is not dependent on the location of workplace and home. The time $t = 0$ corresponds to the time of the first commuter leaves one's workplace.

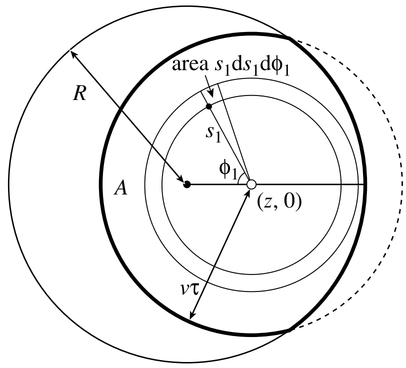

Fig. 3. Small region from which the travel time to the concert hall is s_1/v.

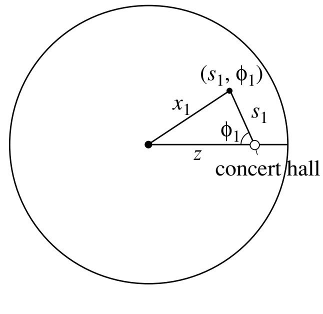

Fig. 4. Transformation of coordinates from (x_1, ψ_1) to (s_1, ϕ_1).

3. Formulation

In this section, we formulate the concert problem. Let us denote the number of potential customers by $n(z, \theta, \tau)$ as a function of the location of a concert hall and the start time. The aim of the concert problem is to find the location and the start time of the concert which maximize this function. If the end time $\tau + c$ exceeds t_h no one can go back home by t_h so $\tau + c$ must be before t_h. This leads that the start time τ must be determined in the range $\tau \in [0, t_h - c]$.

In the following, the objective function $n(z, \theta, \tau)$ is derived. Let us consider the condition that a given commuter be a potential customer. To be a potential customer, the following two conditions have to be met:

(i) Workplace and home of a given commuter must be in region A and region B respectively as shown in Fig. 2.
(ii) This commuter has to leave workplace early enough to be in time for the start time of the concert τ.

The region A is the set of points (workplaces) from which the travel time to the concert hall is within τ, while the region B is the set of points (homes) to which the travel time from the concert hall is within $(t_h - \tau - c)$. By the assumption of constant travel speed, region A represents the intersection of the circle of radius $v\tau$ centered at (z, θ) and the circular city and region B represents the intersection of the circle of radius $v(t_h - \tau - c)$ centered at (z, θ) and the circular city.

Let us consider the condition (i). First, focus on the set of commuters S whose workplaces are within a small region in A with area $s_1 \mathrm{d}s_1 \mathrm{d}\phi_1$ and whose homes are within a small area in B with area $s_2 \mathrm{d}s_2 \mathrm{d}\phi_2$. The number of commuters in S is given by $\rho(s_1, \phi_1, s_2, \phi_2) s_1 s_2 \mathrm{d}s_1 \mathrm{d}\phi_1 \mathrm{d}s_2 \mathrm{d}\phi_2$ by the