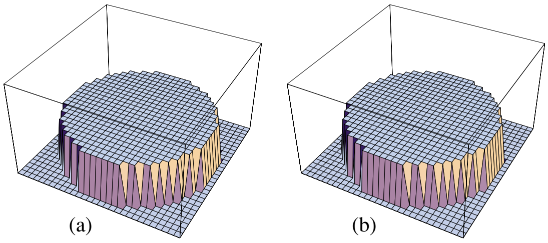

Fig. 7. Distribution of workplaces (a) and homes (b) for case 1.

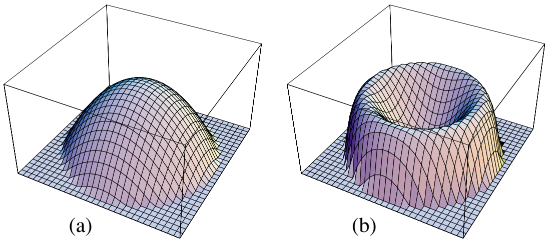

Fig. 8. Distribution of workplaces (a) and homes (b) for case 2.

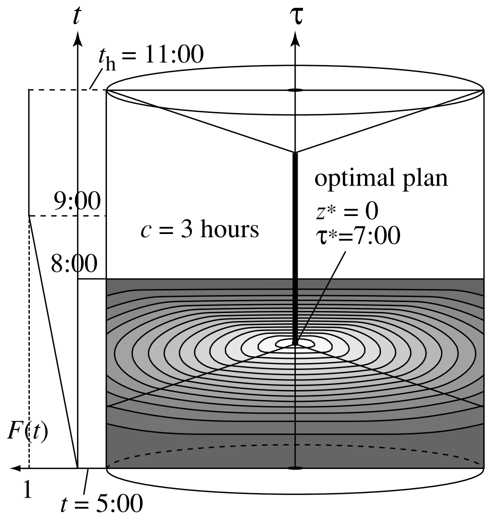

Fig. 9. Contour plot of $n(z, \theta, \tau)$ for case 1.

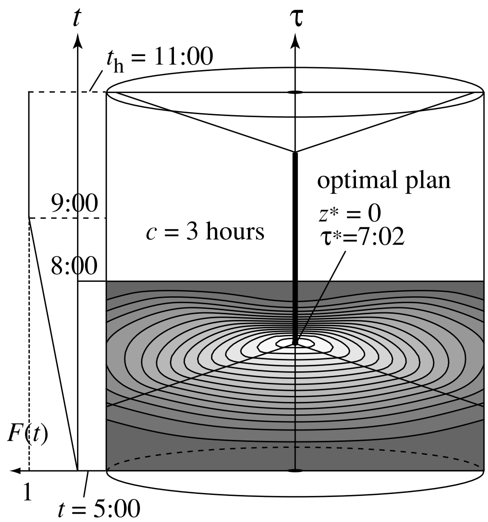

Fig. 10. Contour plot of $n(z, \theta, \tau)$ for case 2.

where N represents the total number of commuters, $\lambda(x_1, \psi_1)$ and $\mu(x_2, \psi_2)$ denote the number of workplaces and homes per unit area at (x_1, ψ_1) and (x_2, ψ_2), respectively. It should be noted that the origin of polar coordinates in Eq. (4) is different from that of Eq. (2); the origin of the former is the city center while that of the latter is the concert hall. In the following analysis, we assume that workplaces and homes are radially sysmetric; densities of workplaces and homes are expressed as a function of the distance from the city center only:

$$\lambda(x_1, \psi_1) = \lambda(x_1), \quad \mu(x_2, \psi_2) = \mu(x_2). \tag{5}$$

To carry out the integration in Eq. (2), trip density must be expressed as a function of $(s_1, \phi_1, s_2, \phi_2)$ instead of $(x_1, \psi_1, x_2, \psi_2)$. As illustrated in Fig. 4, the transformation of coordinates of workplaces from (x_1, ψ_1) to (s_1, ϕ_1) is explained. Using the law of cosines, x_1 can be related to s_1 and ϕ_1 as follows:

$$x_1^2 = s_1^2 + z^2 - 2s_1 z \cos\phi_1. \tag{6}$$

Similarly for homes, x_2 can be related to s_2 and ϕ_2:

$$x_2^2 = s_2^2 + z^2 - 2s_2 z \cos\phi_2. \tag{7}$$

Using these relationship, the densities of workplaces and homes can be written as follows:

$$\lambda(x_1) = \lambda\left(\sqrt{s_1^2 + z^2 - 2s_1 z \cos\phi_1}\right), \tag{8}$$

$$\mu(x_2) = \mu\left(\sqrt{s_2^2 + z^2 - 2s_2 z \cos\phi_2}\right). \tag{9}$$

From the above discussions,

$$\rho(s_1, \phi_1, s_2, \phi_2) = N \cdot \lambda\left(\sqrt{s_1^2 + z^2 - 2s_1 z \cos\phi_1}\right) \cdot \mu\left(\sqrt{s_2^2 + z^2 - 2s_2 z \cos\phi_2}\right). \tag{10}$$

By expressing trip density as a function of $(s_1, \phi_1, s_2, \phi_2)$ using the relationship shown in Eq. (10), $n(z, \theta, \tau)$ can be calculated by carrying out the integration in Eq. (2) in principle. The analytical calculation of the integral, however, is very complicated. Depending on the set of parameter and variable values, the shape of region A and region B varies as illustrated in Figs. 5 and 6. The domain of integration of Eq. (2) is given by the direct product of region A and B. Therefore, there are four shapes of the domain of integration: A-(a) and B-(a), A-(a) and B-(b), A-(b) and B-(a) and A-(b) and B-(b). In Eq. (11), Eq. (12), Eq. (13) and Eq. (14), the expressions of the objective function corresponding to the above four cases are presented. Although it is very difficult to specify which of the four cases occurs for each possible parameter and variable set, the domain of integration can be easily specified given a parameter and variable set. Consequently, the objective function can be calculated by numerical integration.