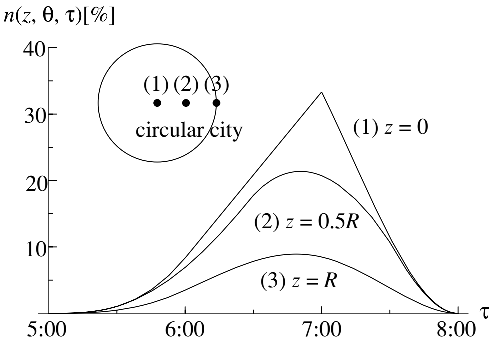

Fig. 11. Plot of $n(z, \theta, \tau)$ as a function of start time τ at three different points in the city for case 1.

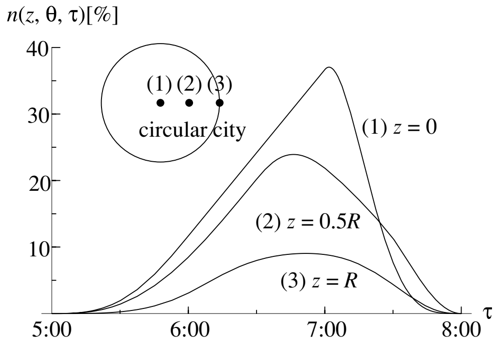

Fig. 12. Plot of $n(z, \theta, \tau)$ as a function of start time τ at three different points in the city for case 2.

A-(a) and B-(a)

$$n(z,\theta,\tau) = 4\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 + 4\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \quad (11)$$

A-(a) and B-(b)

$$n(z,\theta,\tau) = 4\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \quad (12)$$

A-(b) and B-(a)

$$n(z,\theta,\tau) = 4\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 + 4\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 + 4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 + 4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \quad (13)$$

A-(b) and B-(b)

$$n(z,\theta,\tau) = 4\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 + 4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau - \tfrac{s_1}{v}\right) \cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1s_2\mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \quad (14)$$

where

$$s_1^* = z\cos\phi_1 + \sqrt{z^2\cos^2\phi_1 + R^2 - z^2}, \quad \phi_1^* = \arccos\left(\frac{z^2 + v^2\tau^2 - R^2}{2v\tau z}\right), \quad s_2^* = z\cos\phi_2 + \sqrt{z^2\cos^2\phi_2 + R^2 - z^2}, \quad \phi_2^* = \arccos\left(\frac{z^2 + v^2(t_h-\tau-c)^2 - R^2}{2v(t_h-\tau-c)z}\right). \quad (15)$$

5. Numerical Examples

In this section, we first consider the following two radially symmetric models for densities of workplaces and homes.

case 1

$$\lambda(x_1) = \frac{1}{\pi R^2}, \qquad \mu(x_2) = \frac{1}{\pi R^2}, \quad (16)$$

case 2

$$\lambda(x_1) = \frac{2}{\pi R^2}\left(1 - \frac{1}{R^2}x_1^2\right), \quad \mu(x_2) = \frac{6}{\pi R^4}x_2^2\left(1 - \frac{1}{R^2}x_2^2\right). \quad (17)$$

The case 1 is the simplest model assuming that workplaces and homes are uniformly distributed within a city as shown in Fig. 7. The case 2 assumes that workplaces are densely distributed at the center and homes are densely distributed at some distance from the city center.

In the example below, the departure time distribution is given by a linearly increasing function from 5:00 p.m. to 9:00 p.m. as shown in Figs. 9 and 10. We assume the following parameter values: $c = 3$ hours, $2R/v = 2$ hours, $t_h = 11:00$ p.m. Under these assumptions, the objective function $n(z, \theta, \tau)$ is calculated by numerical integration for various values of (z, θ, τ).

Figures 9 and 10 show the contour plot of $n(z, \theta, \tau)$. From these figures, the unique optimal solution can be found. In both cases, the optimal location of the concert hall is at the city center: $z^* = 0$. In the case of Fig. 9, the optimal start time satisfies the distinctive property: the best plan is to start concert as late as possible while satisfying that every commuters attendable to the concert can also go back home by t_h.

Figures 11 and 12 show $n(z, \theta, \tau)$ as a function of τ at three different points in the city. The interesting point to note is that the optimal start time depends upon the location