



Fig. 2. Railway network in Japan.

capacity of facilities in zone  $j$  is given by  $\theta S_j$ . Hence, the most appropriate state is that the following equation is satisfied in every zone:

$$F_j^c = \theta S_j. \quad (6)$$

Since  $F_j^c$  is a function of  $\{S_j\}$ , (6) constitutes  $K$  equations in the  $K$ -variables  $\{S_j\}$ . Therefore, the equilibrium distribution of  $\{S_j\}$  is given as the solution of (6).

## 2.2 Assumptions

In this study, we assume that  $V_{j|i}$  is defined as the summation of (i)  $c_{ij}$ : travel cost between residence zone  $i$  and activity zone  $j$ , and (ii)  $\ln S_j$ : the logarithm of the number of facilities in zone  $j$ :

$$V_{j|i} \{c_{ij}, S_j\} = -ac_{ij} + b \ln S_j, \quad (7)$$

where  $a$  and  $b$  are positive parameters. Generally,  $S_j$  is perceived sensuously, so we assume that  $V_{j|i}$  is in proportion to the logarithm of  $S_j$  which implies the Fechner law (Goldstein, 1989). Meanwhile,  $c_{ij}$  can be perceived rather numerically, then we define that  $V_{j|i}$  is a linear function of  $c_{ij}$ . The validity of preceding assumption have been discussed in detail (Kurita, 2002).

Now, substituting (5) and (7) for (6),

$$S_j = \frac{1}{\theta} F_j^c = \frac{1}{\theta} \sum_{i=1}^K O_i \frac{S_j^\alpha \exp[-\gamma c_{ij}]}{\sum_{j=1}^K S_j^\alpha \exp[-\gamma c_{ij}]} \quad (8)$$

is obtained where  $\alpha = b\lambda$ ,  $\gamma = a\lambda$ . Unfortunately, it is quite tough to solve (8) analytically, so we regard (8) as iterative equations and calculate the equilibrium distribution numerically. The next step in the argument is to discuss the property of distribution.

## 2.3 Property of equilibrium distribution

The model stated above is basically equivalent to the formulation of **Balancing-Mechanism** proposed by Harris and Wilson (1978). Using catastrophe theory, Harris *et al.* analyzed the properties of equilibrium distribution, and clarified the following features:

- [feature 1] equilibrium distribution  $\{S_j\}$  is continuous when  $\alpha \leq 1$ , and discrete when  $\alpha > 1$ .
- [feature 2] highly accumulated zones are emerged when the distribution becomes discrete, and the number of these zones decreases for higher  $\alpha$  and lower  $\gamma$ .

In addition, Rijk and Vorst (1983) derived the following condition about equilibrium distribution:

- [theorem 1] (8) has a unique positive solution  $\{S_j\}$  when  $0 < \alpha \leq 1$ .
- [theorem 2] (8) has at least one equilibrium solution  $\{S_j\}$  when  $\alpha > 1$ .

From these theorems, the existence of equilibrium solution is assured. In this study, we assume uniform distribution as initial distribution, and calculate (8) numerically iteratively. The reason is to clarify the basic feature of model. Though, [theorem 2] pointed out the possibility of the existence of multiple equilibrium distribution, we consider that these analyses are essential.

## 3. Advances of Railway Network in Japan

In this section, we summarize the railway network system in Japan. Since we will apply the preceding model to Japan islands in next section, the discussion stated below is a preparation for analysis in Sec. 4.

### 3.1 Railway Network in Japan

As of January 2008, there are 9,277 stations and 516 lines in Japanese railway network. This railway network puts on all over the whole country, and it is indispensable for our daily life. In this study, we extracted the data of Japanese railway network from “*Ido-Keido tsuki Zenkoku-Ensen Eki Database*” edited by Japan Geographic Data Center (2008). This database summarized the latitude-longitude of all stations and line information such as network structure. Therefore, using the database, we are able to calculate the travel time between two arbitrarily chosen stations. The railway network adopted in the study is shown in Fig. 2.

### 3.2 Construction process and future plans of Shinkansen

*Shinkansen* bullet train is one of the noteworthy projects of Japanese railway network system. It is a representative high-speed transit system in Japan, and promotes mutual interactions in the islands. Total railway length of Shinkansen is over 2,000 km now, and it covers from Hachinohe to Kagoshima. *Mini-Shinkansen* is also a kind of bullet train which is operated on the conventional railway lines. Although, the highest speed of Mini-Shinkansen (130 km/h) is slower than that of Shinkansen (270 km/h), this system frees passenger from a transfer at the station because it directly connect to Shinkansen lines. Yamagata and Akita Shinkansens adopt a Mini-Shinkansen system. Construction process of Shinkansen is summarized in Table 1.

Even now, there are some of plans to construct new lines