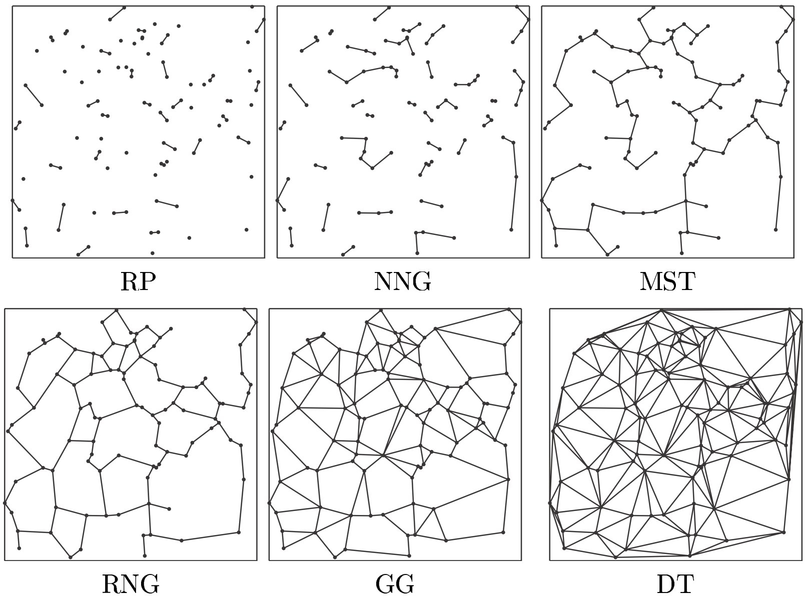

Fig. 1. Proximity graphs on same random points.

contain any other points of P in its interior. We denote it $G_{\mathrm{DT}} = (P, E_{\mathrm{DT}})$. DT is dual graph of Voronoi diagram.

Minimum Spanning Tree (MST) is not proximity graph because MST needs to be global optimum, but related with proximity graphs. MST is defined as the tree which the sum of the Euclidean length of all the edges attains the minimum over all trees. The number of edges is $m_{\mathrm{MST}} = n-1$ because there is no circuit. We denote it $G_{\mathrm{MST}} = (P, E_{\mathrm{MST}})$.

Complete Graph (CG) is obtained by joining two points p_i, p_j of P with an edge if each pair of points has an edge connecting them, and denote it by $G_{\mathrm{CG}} = (P, E_{\mathrm{CG}})$. The number of edges is $m_{\mathrm{CG}} = n(n-1)/2$ because there are pairs of all nodes.

Each proximity graph and its related graphs have following relation: $E_{\mathrm{RP}} \subseteq E_{\mathrm{NNG}} \subseteq E_{\mathrm{MST}} \subseteq E_{\mathrm{RNG}} \subseteq E_{\mathrm{GG}} \subseteq E_{\mathrm{DT}} \subseteq E_{\mathrm{CG}}$. Figure 1 shows these proximity graphs constructed with random 100 points.

3. Evaluation of Graph Configuration

3.1 The length of graph edges with random points in previous works

It has been reported that the estimation of edge length of RP, NNG, GG and DT, but that of RNG and MST are unknown. We estimate those approximately using geometric probability.

We assume a random pattern with theoretical density of points ρ per unit area. Poisson probability law are used to obtain the probability density function of distance from an arbitrary locus to the nearest points. The random variable is denoted by l and a particular value of this distance variable is indicated by L. Let the probability density function of l be $f(l)$, the expectation be μ and the variance be σ^2.

The result of RP is obtained by Pickard (1982) as follows:

$$f_{\mathrm{RP}}(l) = 2\left(\frac{4}{3}\pi + \frac{\sqrt{3}}{2}\right)\rho l \mathrm{e}^{-\left(\frac{4}{3}\pi+\frac{\sqrt{3}}{2}\right)\rho l^2}, \tag{1}$$

$$\mu_{\mathrm{RP}} = \frac{1}{2}\sqrt{\frac{\pi}{\rho(4\pi/3+\sqrt{3}/2)}} \simeq \frac{0.394178}{\sqrt{\rho}},$$

$$\sigma^2_{\mathrm{RP}} = \frac{4-\pi}{4\rho(4\pi/3+\sqrt{3}/2)}.$$

The result of NNG is obtained by Clark and Evans (1954) as follows:

$$f_{\mathrm{NNG}}(l) = 2\rho\pi l \mathrm{e}^{-\rho\pi l^2}, \tag{2}$$

$$\mu_{\mathrm{NNG}} = \frac{1}{2\sqrt{\rho}} = \frac{0.5}{\sqrt{\rho}}, \quad \sigma^2_{\mathrm{NNG}} = \frac{4-\pi}{4\rho\pi}.$$

The result of GG is obtained by Møller (1994) as follows:

$$f_{\mathrm{GG}}(l) = \frac{1}{2}\rho\pi l \mathrm{e}^{-\frac{1}{4}\rho\pi l^2}, \tag{3}$$

$$\mu_{\mathrm{GG}} = \frac{1}{\sqrt{\rho}}, \quad \sigma^2_{\mathrm{GG}} = \frac{4-\pi}{\pi\rho}.$$

The result of DT is obtained by Collins (1968) and Miles (1970) as follows:

$$f_{\mathrm{DT}}(l) = \frac{\pi\rho l}{3}\left\{\sqrt{\rho}l\mathrm{e}^{-\frac{1}{4}\pi\rho l^2} + \mathrm{Erfc}\left(\frac{1}{2}\sqrt{\pi\rho}l\right)\right\}, \tag{4}$$

$$\mu_{\mathrm{DT}} = \frac{32}{9\pi\sqrt{\rho}}, \quad \sigma^2_{\mathrm{DT}} = \frac{5}{\pi\rho} - \frac{1024}{81\pi^2\rho}$$

where Erfc is the complementary error function.

3.2 Estimation of the length of graph edges of RNG and MST

The result of RNG is not obtained and we derive using geomeric probability. At first, we derive the nearest neighbor distance with restricted search region. It has been established that the probability of finding exactly x points in an arbitrary area is given by the Poisson probability law. Let