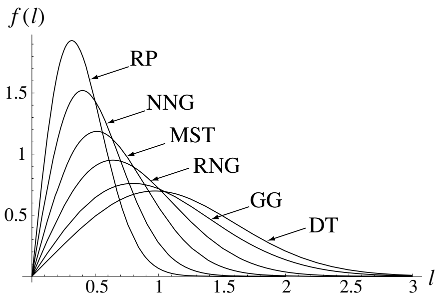


Fig. 4. PDF of edge length of proximity graphs.

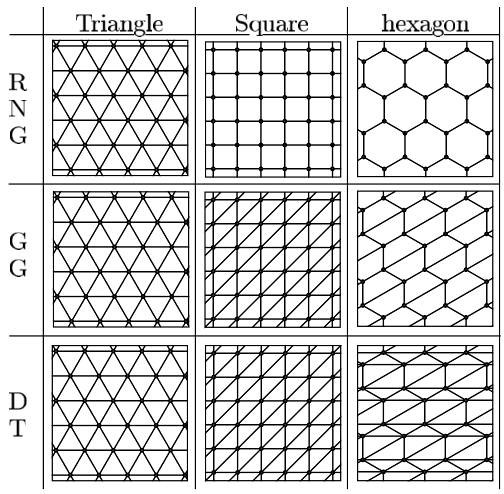


Fig. 5. Proximity graphs with regular points.

Table 1. Total number of graph edges of regular lattice.

| | Triangle | Square | Hexagon |
|---|---|---|---|
| RP | $\frac{1}{2}n$ | $\frac{1}{2}n$ | $\frac{1}{2}n$ |
| NNG | $n$ | $n$ | $n$ |
| MST | $n-1$ | $n-1$ | $n-1$ |
| RNG | $3n$ | $2n$ | $\frac{3}{2}n$ |
| GG | | $3n$ | $2n$ |
| DT | | | $3n$ |

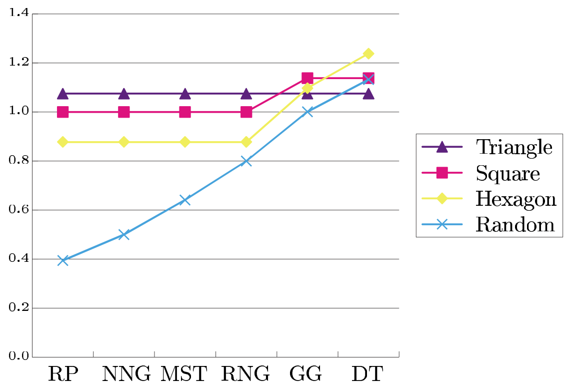


Fig. 6. Average length of graph edges.

We compare the expectation of length with estimations in former researches. As lower bound, Furuyama (2003) estimated $\mu^L_{\mathrm{MST}} = 0.64/\sqrt{\rho}$ from the component percentages of nearest neighbor links using numerical result. As upper bound, Robert (1968) estimated $\mu^U_{\mathrm{MST}} = 0.707/\sqrt{\rho}$. The expectation of length $\mu_{\mathrm{MST}}$ which is derived by this crescent is very close to the lower bound derived by Furuyama and our approximation using the nearest neighbor distance with the restricted search region within crescent is similar to the lower bound of MST.

Figure 4 shows the probabilistic density functions with $\rho$=1. As the search region become smaller from RP to DT, the length of edges become longer because the number of connectable points increase. As the distribution of edge length shifts to the right from RP to DT, the expectation and the variance also become larger. Especially, the functions of RNG and MST which are derived in this section lie between that of NNG and GG.

### 3.3 Comparison with the length of graph edges with regular points

Figure 5 shows the proximity graphs with regular points of triangular lattice, square lattice and hexagonal lattice. If the node is degenerated, we include the edges. As you can see, RNG can construct typical grid road network on each lattice.

In the region which contain $n$ points in area $S$, we can easily calculate the total number and length of edges if we don't consider the condition of the boundary. Table 1 shows the total number of edges, and Table 2 shows the total length of edges

By deviding total length of edges by total number of edges, we get the average length of edge. From the result of random point in previous section, Table 3 shows the result of the average length of graph edges in both regular point and random point. Figure 6 shows the numerical result of Table 3 with $\rho = 1$. On random point, the average length of edge gets longer from RP to DT, and is less than that of regular lattice except DT. On triangle lattice and square lattice, the average length of edges hardly changes from RP to DT. On hexagonal lattice, the average length of edges hardly changes from RP to RNG, but get longer from GG to DT.

## 4. Evaluation of Travel Efficiency on Graphs

### 4.1 Model description

In this section, we analyze the efficiecy of travel on the graphs to compare with some routing system of ideal road network pattern. Most of transportation model treat discrete network or continuous plane. On continuous plane, there are several kind of theoretical distance like Euclidean distance and rectilinear distance. We call the set of edges *network* when the edges have attribution like distance.

We assume a random pattern with theoretical density of points $\rho$ per unit area. There are $n$ demand points in rectangular region whose side is $a$ and area is $S = a^2$. The trip demand uniformly and independently distributed between two points and the total of trips is $n(n-1)$.

Types of distance are Euclidean distance $u$, rectilinear distance $r$ and network distance $d$, and "the travel distance" and "the ratio to Euclidean distance" are compared.

The distance of two point between $p_i = (x_i, y_i)$ and