where N is the number of oscillators and $\mathbf{P_c}(\mathbf{x}_i(t), \mathbf{x}_j(t))$ denotes the coupling between the oscillators. ϵ_{ij} is a newly introduced parameter expressing the coupling strength between the ith and jth oscillators, where $N^{-1}\sum_{j=1}^{N}\epsilon_{ij}$ is assumed to be sufficiently smaller than unity. $\mathbf{F}_i(\mathbf{x}_i)$ denotes a set of functions describing a limit cycle. We assume that the frequencies of the oscillators are slightly different from each other in nature with the magnitude of the difference being characterized by ϵ_d that is smaller than $N^{-1}\sum_{j=1}^{N}\epsilon_{ij}$. Then, $\mathbf{F}_i(\mathbf{x}_i)$ is divided into a part common to all the oscillators and the deviation from it as $\mathbf{F}_i(\mathbf{x}_i) = \mathbf{F}(\mathbf{x}_i) + \epsilon_d \mathbf{f}_i(\mathbf{x}_i)$ (we assume that $\mathbf{F}(\mathbf{x}_i)$, $\mathbf{f}_i(\mathbf{x}_i)$, and $\mathbf{P_c}(\mathbf{x}_i(t), \mathbf{x}_j(t))$ are the functions of $O(1)$). Equation (1) is generally reduced to a phase model as (Kuramoto, 1984)

$$\dot{\phi}_i = \bar{\omega} + \epsilon_d\omega_i + \frac{1}{N}\sum_{j=1}^{N}\epsilon_{ij}q_c(\phi_i(t) - \phi_j(t)), \quad (2)$$

where

$$\omega_i = \frac{1}{2\pi}\int_0^{2\pi} d\theta \mathbf{Z}(\phi_i + \theta)\cdot \mathbf{f}_i(\mathbf{x}_0(\phi_i + \theta)), \quad (3)$$

and

$$q_c(\phi_i - \phi_j) = \frac{1}{2\pi}\int_0^{2\pi} d\theta \mathbf{Z}(\phi_i + \theta)\cdot \mathbf{P_c}(\mathbf{x}_0(\phi_i + \theta), \mathbf{x}_0(\phi_j + \theta)). \quad (4)$$

Here, $\mathbf{x}_0(\phi)$ denotes a point on the limit cycle at a phase ϕ, and $\bar{\omega}$ denotes the increasing rate of the phase, when the inhomogeneity $\epsilon_d \mathbf{f}_i(\mathbf{x}_i)$ and the coupling between oscillators are absent. Since the limit cycle constitutes a closed orbit, $\mathbf{x}_0(\phi) = \mathbf{x}_0(\phi + 2\pi)$ is naturally satisfied. $\mathbf{Z}(\phi) \equiv (\mathrm{grad}_{\mathbf{x}}\phi)_{\mathbf{x}=\mathbf{x}_0(\phi)}$ is called phase response function. It is noted that $|\mathbf{Z}(\phi)|$ should not be extremely large for any ϕ because the phase description is valid only when $\phi - \bar{\omega}t$ is kept almost constant during an oscillation period (Kuramoto, 1984). $q_c(\phi_i(t) - \phi_j(t))$ is called coupling function, whose functional form can be experimentally derived either by specifying the phase response function $\mathbf{Z}(\phi)$ (if the interaction $\mathbf{P_c}(\mathbf{x}_i(t), \mathbf{x}_j(t))$ is already known) (Kiss *et al.*, 2005), or by analyzing the period of one of two-coupled oscillators when they are not completely synchronized (Miyazaki and Kinoshita, 2006a, b). $q_c(\phi_i(t) - \phi_j(t))$ is expanded to Fourier series as $q_c(\phi_i(t) - \phi_j(t)) = \sum_k a_k^{(c)}\exp[ik(\phi_i(t) - \phi_j(t))]$, where $a_{-k}^{(c)} = a_k^{(c)*}$ should be satisfied.

We consider a case where several measurement and stimulation nodes are placed in the system, as shown in Fig. 1. Here, we have called an element used for the measurement of the outputs from its neighborhood oscillators as "measurement node", while that used for the stimulation of the feedback signals to its neighborhood oscillators as "stimulation node". The data obtained from the measurement nodes are analyzed at the host computer and the feedback signals with time delays are applied from the stimulation nodes to the oscillators. Thus, the observables and the applied feedback signals are not uniform, in contrast to our previous work where they are uniform (Kano and Kinoshita, 2008).

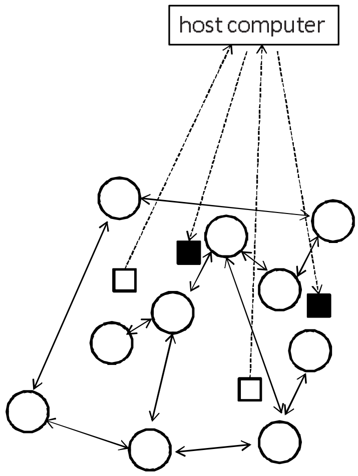

Fig. 1. Scheme of a system considered in the theory. The oscillators (empty circles) are coupled to each other ununiformly (left right arrows). The data obtained from several measurement nodes (empty squares) are analyzed at the host computer and the feedback signals with time delays are applied from the stimulation nodes (filled squares).

The model equation is then given in the following way:

$$\dot{\mathbf{x}}_i = \mathbf{F}_i(\mathbf{x}_i) + \frac{1}{N}\sum_{j=1}^{N}\epsilon_{ij}\mathbf{P_c}(\mathbf{x}_i(t), \mathbf{x}_j(t)) + \frac{1}{N}\sum_{\beta,\gamma}\epsilon'_{\beta\gamma}\rho_i^{(\beta)}\sum_{m=1}^{2M+1}\Gamma_m P_0^{(\gamma)}(t - \tau_m)\mathbf{r}, \quad (5)$$

where β and γ denote indices of the stimulation and measurement nodes, respectively. $\epsilon'_{\beta\gamma}$ characterizes the rate of the output from the γth measurement node to the input to the βth stimulation node. $P_0^{(\gamma)}(t) \equiv \sum_{j=1}^{N}\sigma_j^{(\gamma)}p(\mathbf{x}_i(t))$ is the output from the γth node, where $p(\mathbf{x}_j(t))$ is an arbitrary single-valued function of $\mathbf{x}_j(t)$, and $\sigma_j^{(\gamma)}$ is a weighting factor for the measurement through the γth node. $\rho_i^{(\beta)}$ characterizes the magnitude of the feedback signal applied from the βth node to the ith oscillator. τ_m and Γ_m are the time delay and strength of the mth signal, respectively, which we will specify in the following. $\mathbf{r}$ is a unit vector whose dimension is equal to that of $\mathbf{x}_i$, and it can be selected in an arbitrary manner. The number of the feedback signals are set at $2M + 1$, where the definition of M will be described later.

Now we assume that the contribution of the third term in the right-hand side of Eq. (5) is sufficiently smaller than that of $\mathbf{F}_i(\mathbf{x}_i)$. Then, Eq. (5) is reduced to the phase model as

$$\dot{\phi}_i = \bar{\omega} + \epsilon_d\omega_i + \frac{1}{N}\sum_{j=1}^{N}\epsilon_{ij}q_c(\phi_i(t) - \phi_j(t)) + \frac{1}{N}\sum_{\beta,\gamma}\epsilon'_{\beta\gamma}\rho_i^{(\beta)} \cdot \sum_{m=1}^{2M+1}\Gamma_m\sum_{j=1}^{N}\sigma_j^{(\gamma)}q_f(\phi_i(t) - \phi_j(t - \tau_m)), \quad (6)$$