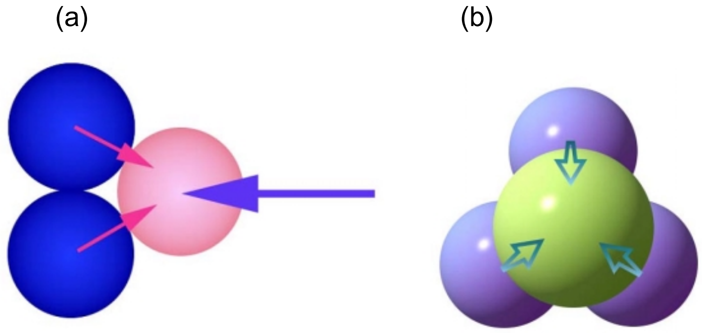

Fig. 5. Stable arrangements of cells in (a) 2D and (b) 3D cases. A force acted on a sphere with light color is balanced by the resistive forces by the spheres with dark color.

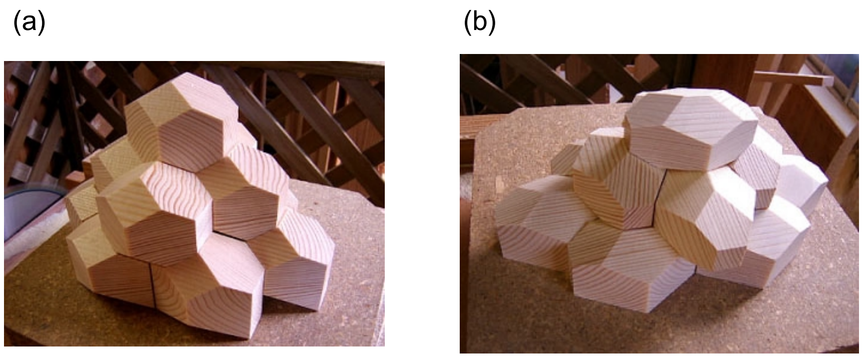

Fig. 6. Models of two 14-hedra for stable space division hand-made by Hiroshi Nakagawa. (a) Kelvin's 14-hedra, (b) Williams' 14-hedra.

in the four-leg problem in the basic mechaqnics, where the forces at the bottom of four legs of a table touching the floor cannot be determined uniquely.

As one of polyhedra whose equal groups fill the space, the rhombic dodecahedron and the truncated octahedron is widely known in addition to the cube. The rhombic dodecahedron is a Voronoi polyhedron from the point arrangement with face-centered cubic lattice, while the truncated octahedron is the Voronoi polyhedron from the body-centered cubic lattice. The rhombic dodecahedron has four vertices where four edges meet, and in the space division by rhombic dodecahedra six polyhedra gather at these vertices. Therefore, this space division is not a stable one. On the other hand, the space division by the truncated octahedra is stable, since in this case always four polyhedra gather at each vertex.

4. Pentagon Face in Space Division

In real space divisions the faces are often curved planes. Here, it is assumed that faces and edges of polyhedra in space division can be curved. In the following we denote the numbers of faces, edges and vertices by f, e, v, respectively. For a polyhedron taken out from a stable space division the numbers e and v satisfy the relation

$$2e = 3v, \tag{1}$$

since three edges gather at a vertex and an edge connects with two vertices. On the other hand, the Euler's theorem gives

$$v - e + f = 2. \tag{2}$$

From these equations we have

$$v = 2(f - 2), e = 3(f - 2). \tag{3}$$

For a polyhedron with $f = 14$, we have $v = 24$ and $e = 36$.

Shapes of faces of a 14-hedron cannot be uniquely determined, and it is assumed here that a face has p edges on the average. Since one edge is shared by two faces, we have $e = pf/2$, hence

$$p = 2e/f = 5.14\ldots. \tag{4}$$

This result means that the distribution of types of polygons should have a peak around the value 5, and agrees with observations that the highest frequency for pentagons in the space division. For example, according to a research of metallic glasses, the distribution of p showed $p = 5$ (40%), $p = 6$ (30%), $p = 4$ (20%), and an average number of edges per a face was 5.12 (Lines, 1994).

One of the important space divisions is that by soap bubbles, where the minimal surface principle is working and the total area of faces gives a criterion for selection of space division pattern. Let the surface area and the volume of a polyhedron be denoted by S and V, respectively. Then, a good parameter of the criterion is the nondimensional ratio S^3/V^2 or its cubic root,

$$c = (S^3/V^2)^{1/3}. \tag{5}$$

The minimal surface principle requires smaller value of c. In the following the shapes of faces are examined for some examples based on this principle.