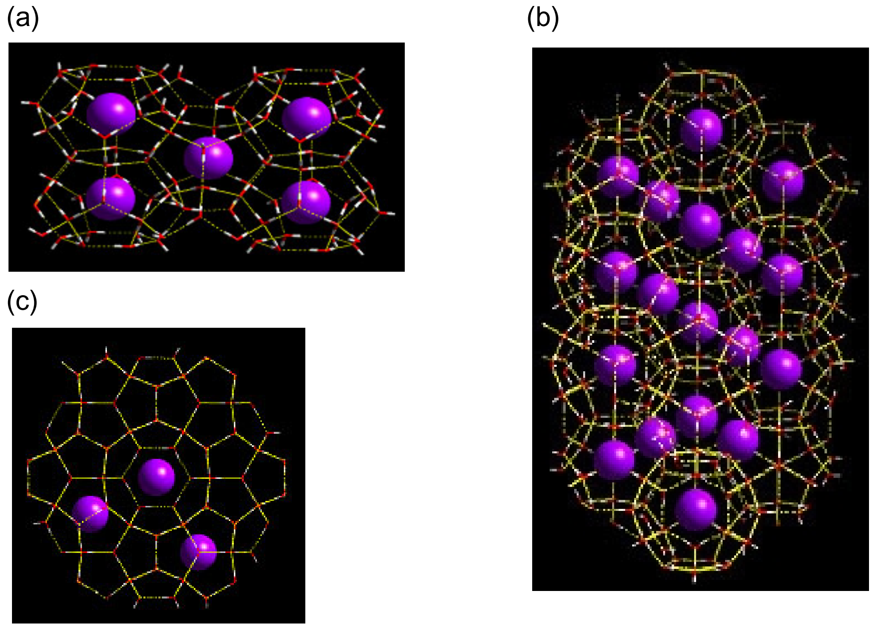

Fig. 9. Clathrate hydrates are formed when water molecules arrange themselves in a cage-like structure around small molecules. (a) Structure I, whose unit cell consists of cage 5^{12} and cage $5^{12}6^2$, (b) structure II, (c) structure H. (Provided by Belosludov (Belosludov *et al.*, 2007).)

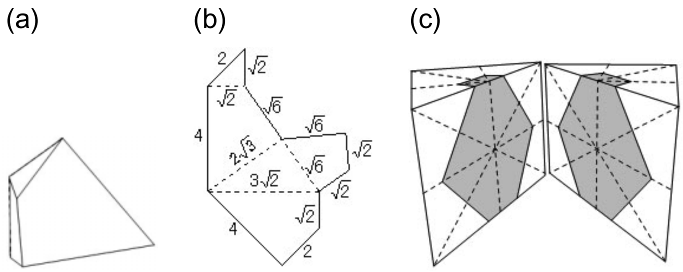

Fig. 10. (a) Pentahedron as an element building Federov's polyhedra, (b) the development of this pentahedron, where edge lengths are shown, (c) construction of a cube with edge length 8 by packing 96 elements of the pentahedron (the gray part indicates the truncated octahedron).

hydrate, as shown in Table 1, among which the last three are found universally and called strucures I, II and H, respectively (Fig. 9). The clathrate hydrate corresponding to the Williams' 14-hedron was recently discovered (Manakov, 2002, 2004) and exists in a special physical environment.

It is worth noting that all of the three clathrate hydrates in this table have a high fraction of pentagons. One answer to the question "why the pentagon plays a dominant role in complex polyhedral structure?" is given by the average number of edges in stable packing of included polygons (Eq. (4)). An easy way of convincing ourselves of this answer would be to consider a process to bend a finite plane sheet to a closed surface. Assume that a molecular structure forming a sheet has the graphite structure, which is an arrangement of equal hexagons. In order to form a closed surface from graphite structure we need to exchange some hexagons with simpler polygons, otherwise the Euler's theorem is not satisfied. In order to produce a smooth closed surface, it will be natural to choose pentagons rather than quadrilaterals or triangles. The term "natural" means the less amount of bending energy.

5. A New Fact about Parallel Polyhedra

Before describing on a new fact, a recent episode suggesting the meaning of this fact is introduced. In June 2008 an international congress was held in Moscow for celebrating the 100 years anniversary of birth of the Russian mathematician L. S. Pontrjagin. In this congress a Japanese mathematician J. Akiyama made a lecture on the set and the element number of regular polyhedra (Akiyama, 2008), where he talked on the development patterns of the regular tetrahedron (Akiyama, 2007). If one cuts a paper-made tetrahedron by scissors to a planar development of any shape (not necessarily cutting along the edges of the tetrahedron), one can fill the plane precisely with this development and its congruent copies. In this sense the tetrahedron is looked upon as a "planar-tessellation producer". Any tetrahedron made of four congruent triangular faces has this property.

Now, let us consider a problem "what is a space-tessellation producer?", i.e. "what is an elementary body producing space filling bodies?". General solution is quite difficult, but simple examples are found as follows (Akiyama, 2009). One can construct all of the parallel polyhedra with one kind of element as shown in Figs. 10(a) and (b), where its mirror image is looked upon as the same one.