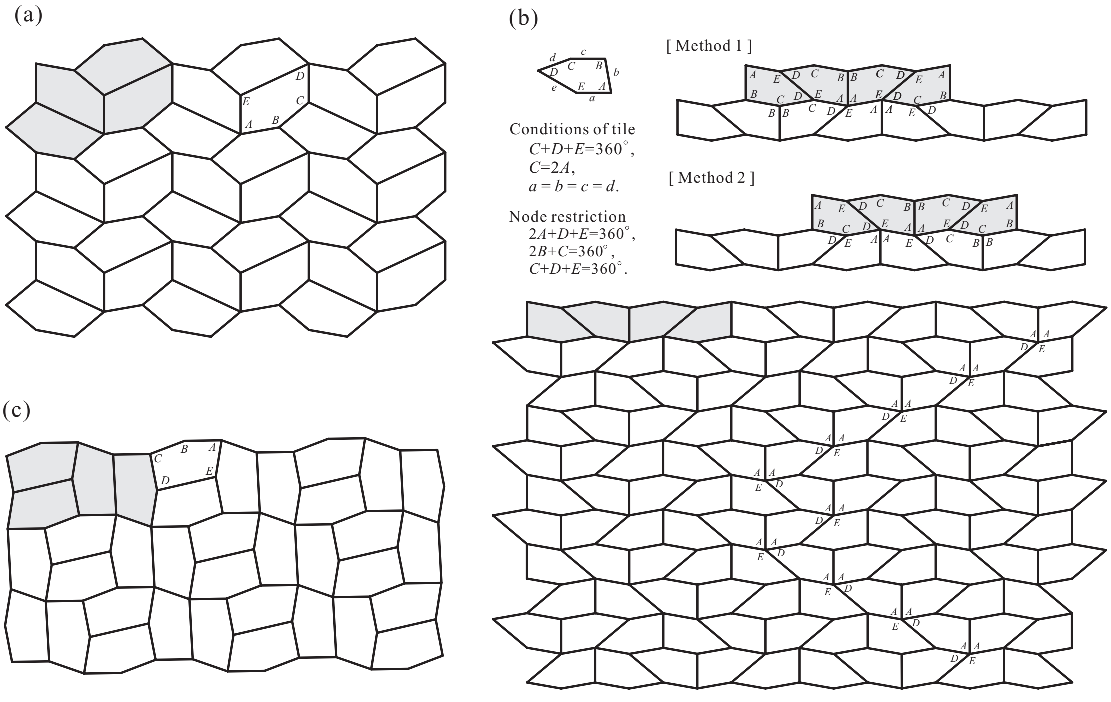


Fig. 5. Tilings of C11. (a) C11-T1A. (b) C11-T1C. (c) C11-T2.

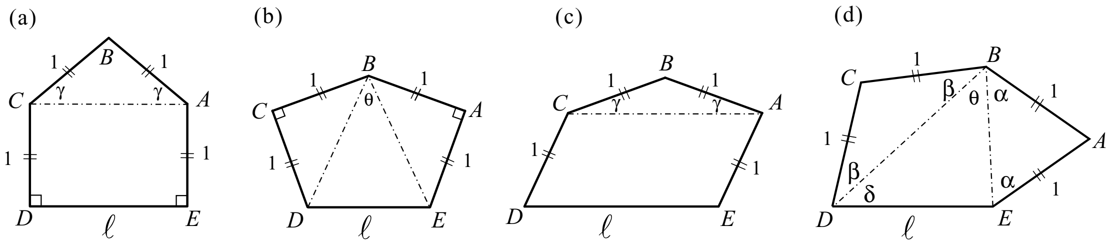


Fig. 6. Relationship between internal angles of tiles. (a) C22-T1A. (b) C22-T2&4. (c) C20-T1A. (d) Other.

dition of $a = d$ and C20-T2 tiles satisfying only $a = c$, $b = d$ both allow only periodic tiling using the smallest fundamental region.

As shown in Fig. 2(c), C20-T1A tiles satisfying $a = d$, $b = c$ allow multipatterned tiling with $DE$-regular and $DE$-reversed mixed without breaking the node restriction.

C22-T1A tiles in Fig. 1(a) and C22-T1E tiles in Fig. 1(b) allow periodic tiling using the smallest fundamental region (the pale gray region) as well as multipatterned tiling by using the smallest fundamental region and the smallest fundamental region rotated by 180° or mirrored (the dark gray region). As each figure shows, the smallest fundamental region rotated by 180° has the same shape as the smallest fundamental region mirrored. In the C22-T1A tiling pattern in Fig. 1(a), the pentagon is symmetrical to the axis passing through vertex $B$ and edge $DE$. Additionally, the smallest fundamental region is symmetrical. Therefore, all patterns can be considered to be the same if the vertex sign is not entered. On the other hand, two C22-T1E tiles bonded on the edge $d$ ($CD$-reversed) are a hexagon. If we consider that the element of tiling is the hexagonal shape, we can see that tiling is the always same pattern.

C22-T2&4 tiles forming the tiling pattern in Fig. 1(c) cannot allow multipatterned tiling under the node restriction. The pentagon is symmetrical to the axis passing through the midpoint between vertex $B$ and edge $DE$. Therefore, C22-T2&4 tiles can form the tiling pattern as in Fig. 1(c) without observing the node restriction. (As mentioned above, we can form the tiling with the nodes used in type 4 because C22-T2&4 tiles also belong to type 4.) They all have the same pattern unless tile vertex signs are entered.

As shown in Fig. 5(b), the C11-T1C tile has two methods for combining the smallest fundamental regions through translation. Therefore, C11-T1C tiles allow multipatterned tiling only with the translation operation. However, the C11-T1C tile consists of three congruent isosceles triangles: $ABE$, $CEB$, and $CDE$. If we consider that tiling is formed of the smallest fundamental region composed of four bonded congruent isosceles triangles with different directions, a tiling is periodic with the same pattern at all times.

Figure 4(b) C12-T7, (c) C12-T8, and (d) C12-T9 correspond to the idiomatic expressions of type 7, type 8, and type 9, respectively. The tilings with C12-T1C in Fig. 4(a), C11-T1A in Fig. 5(a), and C11-T2 in Fig 5(c) are also shown by Marjorie Rice (Schattschneider, 1978, 1981;