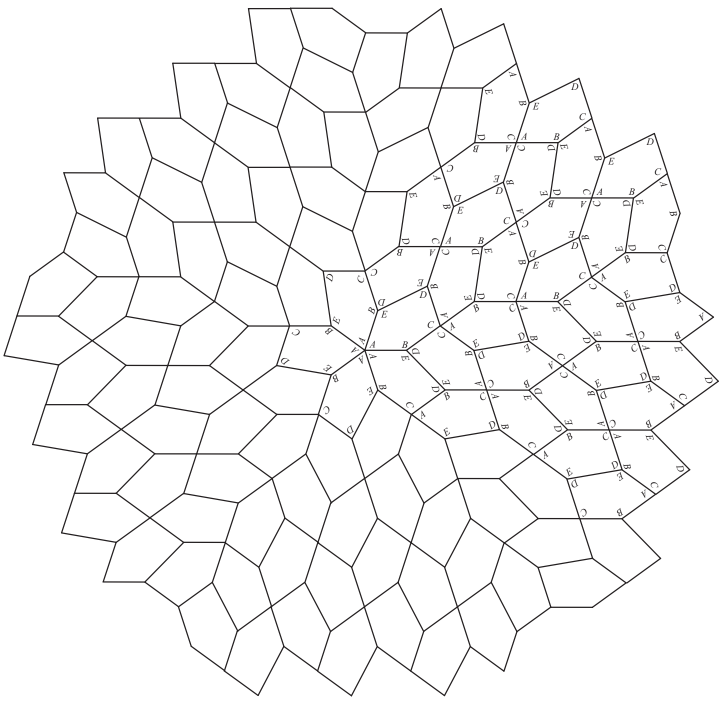

Fig. 9. Special tiling by convex pentagons of C20-T2 with $A = 72^\circ$

"$AADE$, BDE, CCB," just as C11-T2 and also tiling using nodes "$CCAA$, BDE" as a type 2 (pentagonal tile that satisfies $B + D + E = 360^\circ$). Furthermore, the multipatterned tiling of number 13 in Table 4 is generated because both nodes were used simultaneously in the tiling.

At the conclusion of this section, we introduce a special tiling using convex pentagons with four equal-length edges of C20-T2 and C11-T2 with $A = 72^\circ$. As shown in Fig. 9, the tiling has only one 5-valent node in the tiling pattern. This tiling always uses the DE-reversed. One kind of 5-valent $AAAAA$ node, one kind of 4-valent $AACC$ node, and one kind of 3-valent BDE node are used. What has no periodicity, however, is a tiling of 5-rotation symmetry. The tiling in Fig. 9 is a special case in which a 5-valent node appears only in one center even if the tiling is continued beyond the illustration. In terms of maximum and finite tiling, the 5-valent node, which can be ignored on its own, and tiling in Fig. 9 can be considered to satisfy the relationship of $V_3 : V_4 \approx 2 : 1$ (Sugimoto and Ogawa, 2006b). Note that, based on Table 3, C20-T2 tiles reserve freedom θ only by assuming $A = 72^\circ$ ($\alpha = 54^\circ$). C11-T2 tiles with $A = 72^\circ$ fall under C20-T2 tiles satisfying $\alpha = 54^\circ$, $\theta = 54^\circ$. Additionally, a C20-T2 tile becomes an equilateral pentagon when $\alpha = 54^\circ$, $\theta = \cos^{-1}(3/(4\sin(2\pi/5))) \approx 37.945^\circ$. The tiling in Fig. 9 is also enabled by using this equilateral pentagon (Hirschhorn and Hunt, 1985).

5. Discussion of Convex Pentagons with Four Equal-length Edges Eliminated by Topological Judgment but Confirmed by the Geometric Judgment

In Report I, we discussed 33 cases in Tables 3 and 4 of Report I by (i) topological judgment (graph theory) to investigate the possibility of tiling using symbolized notation without breaking down the order of pentagonal meshes, and (ii) geometric judgment to investigate the possibility of the existence of the convex pentagon in Euclidean space. As a result, according to Report I, the convex pentagons of DE-regular 1–5, 8 in Table 3 and DE-reversed 1, 8 in Table 4 are eliminated by the topological judgment. But they are confirmed by the geometric judgment (i.e., the convex pentagons exist though their tilings are impossible under the node restriction). Here, we correct the results of Report I. In Table 7 of Report I, we classified DE-regular 13, 17, 18 into "N" of the geometric judgment. But the classification was a mistake. The cases of DE-regular 13, 17, 18 are confirmed by the geometric judgment and are eliminated by the topological judgment. Therefore, in this section, we consider the convex pentagons of DE-regular 1–5, 8, 13, 17, 18 in table 3 and DE-reversed 1, 8 in Table 4 of Report I.

Tables 5 and 6 summarize the properties of convex pentagons with four equal-length edges of DE-regular 1–5, 8, 13, 17, 18 and DE-reversed 1, 8. In addition, they show the probability of tiling without the node restriction (Sugimoto and Ogawa, 2006b).

Table 5's column of node restriction that was applied in Report I shows 3- and 4-valent nodes of DE-regular 1–5, 8, 13, 17, 18 and DE-reversed 1, 8. They are mentioned in Tables 3 and 4 of Report I. The next column shows the necessary and sufficient conditions that convex pentagons with four equal-length edges should satisfy so that the relationship of the internal angles of each node restriction is realized. Based on the conditions, DE-regular 2 and DE-reversed 8 convex pentagons with four equal-length edges are the same as Table 2's C22-T1A and C12-T1C tiles, respectively. Therefore, DE-regular 2 and DE-reversed 8 convex pentagons are pentagonal tiles even though they do