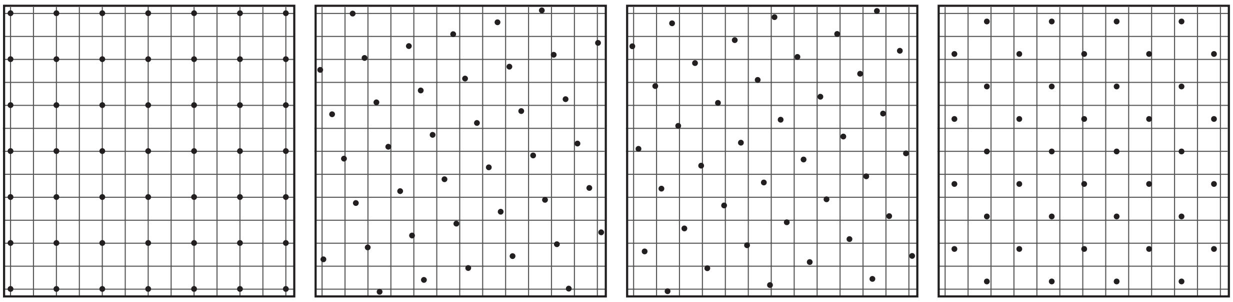

(a) $\theta=0$ (b) $\theta=\frac{\pi}{12}$ (c) $\theta=\frac{\pi}{6}$ (d) $\theta=\frac{\pi}{4}$

Fig. 1. Rotated regular point patterns.

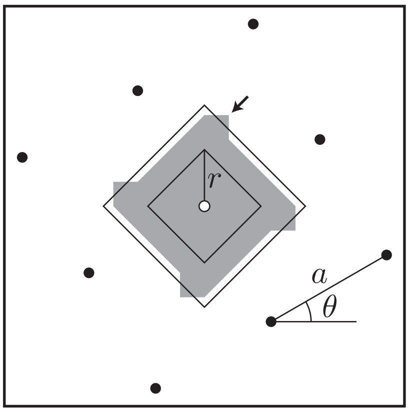

Fig. 2. Region where the white point is the nearest.

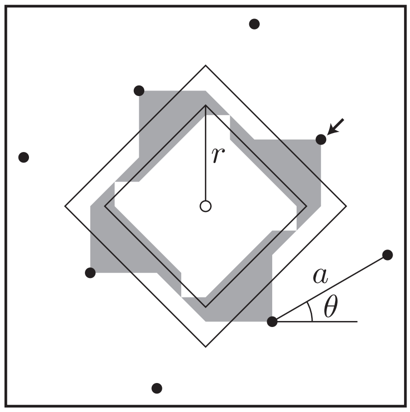

Fig. 3. Region where the white point is the second nearest.

a facility location problem in which customers are serviced by the nearest or the second nearest facility.

A number of facility location models that explicitly include the possibility of closing have been proposed. Weaver and Church (1985) addressed the vector assignment p-median problem, where a certain percentage of customers could be serviced by the kth nearest facility. Pirkul (1989) studied a similar problem in which customers are served by two facilities designated as primary and secondary facilities. Drezner (1987) generalized the p-median and p-center problems by considering the possibility that some of the facilities become inactive. Berman *et al.* (2007) extended Drezner's work and demonstrated that the probability of facility failure has a strong effect on the optimal facility location. Snyder and Daskin (2005) presented two reliability models based on the p-median problem and the uncapacitated fixed-charge location problem. Church *et al.* (2004) developed a interdiction model to identify the set of facilities that, if interdicted, causes the greatest loss. Church and Scaparra (2007) extended the model to generate the range of possible failures and impacts. A survey of facility location problems under uncertainty is provided in Snyder (2006).

Most of the previous studies concerning facility location problems with closing of facilities used discrete network models, in which demand occurs only at nodes of a network. Since discrete models can easily describe realistic situations, the focus is on developing algorithms and obtaining numerical solutions. This paper, in contrast, uses a continuous model, in which demand occurs anywhere on a plane. Continuous models often yield simple closed form solutions, which provide fundamental relationships between variables.

The rest of this paper is organized as follows. The next section derives the distributions of the nearest and the second nearest rectilinear distances for the rotated regular patterns. Section 3 presents an application to a facility location problem. The final section summarizes our main results.

2. Nearest and Second Nearest Distance Distributions

Let R_1, R_2 be the rectilinear distances from an arbitrary location in a study region to the nearest and the second nearest points. Let $f_1(r)$, $f_2(r)$ be the probability density functions of R_1, R_2. We call $f_1(r)$, $f_2(r)$ the nearest distance distribution and the second nearest distance distribution, respectively. In this section, we derive $f_1(r)$, $f_2(r)$ for the rotated regular patterns.

Let $F_1(r)$ be the cumulative distribution function of the nearest distance R_1. $F_1(r)$ is the probability that $R_1 \le r$, which is given by

$$F_1(r) = \frac{S_1(r)}{S} \tag{2}$$

where S and $S_1(r)$ are the area of the study region and the area of the region such that $R_1 \le r$ in the study region, respectively. Differentiating Eq. (2) with respect to r yields