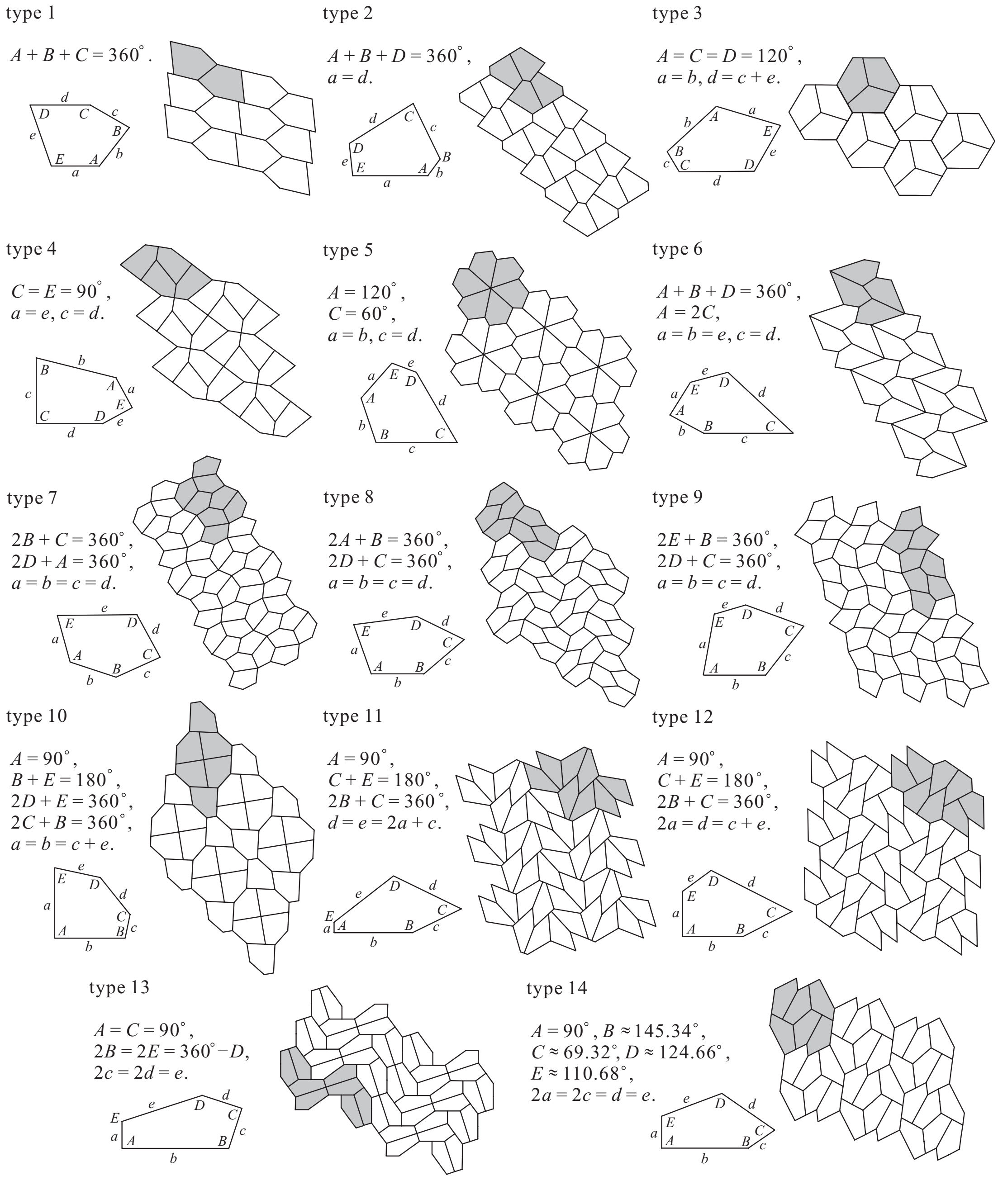

Fig. 1. Convex pentagonal tiles of type 1–type 14.

tagons introduced in Proposition 1 in this paper may not be sufficient for solving the convex pentagonal tiling problem. However, by accumulating such properties one by one, we are steadily approaching the complete solution of the problem.

2. Proof of Proposition 1

First, the definition required for proof is introduced. Given a normal tiling $\mathcal{J}_0$ by polygons, let W be a closed disk of radius $\rho(>0)$ on the plane. Then, let F_1 and F_2 denote the set of the polygons contained in W and the set of polygons meeting the boundary of W but not contained in W, respectively. Here, we define $F := F_1 \cup F_2$ and denote by $P(F)$ the number of polygons in F. In addition, let $E(F)$ and $N(F)$ denote the numbers of edges and nodes of the tiling in F, respectively. The tiling $\mathcal{J}_0$ is balanced if it is normal and satisfies the following condition: the limits $\lim_{\rho\to\infty} N(F)/P(F)$ and $\lim_{\rho\to\infty} E(F)/P(F)$ exist and are finite (see Section 3.3 in Grünbaum and Shephard (1987)).

Proof. Given a closed disk W of radius $\rho(>0)$ on normal tiling $\mathcal{J}$, let $P(F_1)$, $P(F_2)$, and $P(F)$ denote the number of pentagons in F_1, the number of pentagons in F_2, and the number of pentagons in F, respectively. Since $\mathcal{J}$ is normal, the following relations hold (see Statement 3.2.2 in Grünbaum and Shephard (1987)):

$$\lim_{\rho\to\infty}\frac{P(F_1)}{P(F)}=1 \text{ and } \lim_{\rho\to\infty}\frac{P(F_2)}{P(F)}=0. \qquad (1)$$

Two tiles are called adjacent if they have an edge in