

Fig. 5. $(X_0^{(1)}, X_0^{(2)})$ for a = 3.8 and D = 0.433 in Eq. (24) are plotted, when $|X_n^{(1)} - X_n^{(2)}|$ becomes smaller than the threshold $l_c = 10^{-3}$ within 20 steps in (a). (b) is a blowup of (a).



Fig. 6. Relaxation time *n* satisfying $|\psi_n^{(2)} - \psi_n^{(1)}| < 10^{-3}$ as a first passage time of initial points $(\Re(\psi_0^{(2)}), \Im(\psi_0^{(2)}))$ to the complete chaos synchronization are plotted in a gray scale for a = 2.56 and K = 0.6 in (a), a = 2.52 and K = 0.55 in (b).

4. Relaxation Times and Complex Basin Structure

For the logistic map f(x) = ax(1 - x), a bidirectionally coupled system consisting of two identical chaotic oscillators $X^{(1)}$ and $X^{(2)}$

$$\begin{cases} X_{n+1}^{(1)} = f(X_n^{(1)}) + K[f(X_n^{(2)}) - f(X_n^{(1)})], \\ X_{n+1}^{(2)} = f(X_n^{(2)}) + K[f(X_n^{(1)}) - f(X_n^{(2)})] \end{cases}$$
(24)

is considered, where *K* and *D* respectively denote the coupling strength and the largest Lyapunov exponent of the logistic map with $K = (1 + \exp(-D))/2$. For large enough *K*, the complete chaos synchronization occurs (Fujisaka and Yamada, 1983; Yamada and Fujisaka, 1983). In Fig. 5, initial points $(X_0^{(1)}, X_0^{(2)})$ for a = 3.8 and D = 0.433 are

plotted, when the difference $|X_n^{(1)} - X_n^{(2)}|$ becomes smaller than the threshold $l_c = 10^{-3}$ within 20 steps with numerical iterations of Eq. (24) (this result has not reported in any original paper, but first published in the following tutorial paper: Fujisaka *et al.*, 1996). Relaxation times to an attractor of the complete chaos synchronization are found to depend on the initial condition in the phase space in a complex and self-similar way, which is similar to riddled basin structure with multiple attractors (Alexander *et al.*, 1992; Ott *et al.*, 1994). However, it should be noted that the complete chaos synchronization of our system has a single attractor.

For a unidirectionally coupled system consisting of the driving system $\psi^{(1)}$ and the response system $\psi^{(2)}$, complete chaos synchronization is achieved by changing the coupling strength. In Fig. 6, we plot relaxation times of initial points