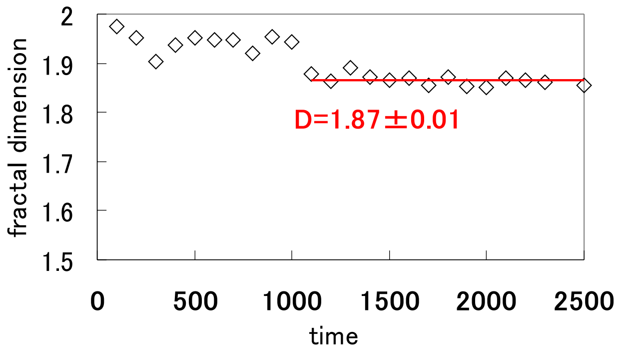

Fig. 3. Time variation of the fractal dimension for $S_0 = 0.0001$. The average value of the fractal dimension for $t \geq 1100$ is 1.87.

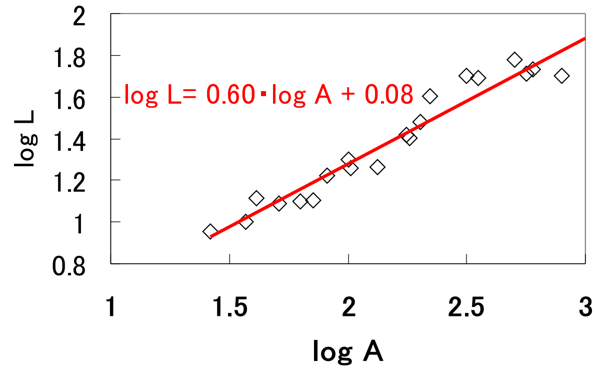

Fig. 5. Relationship between drainage area A and stream length L. This result shows the Hack's law.

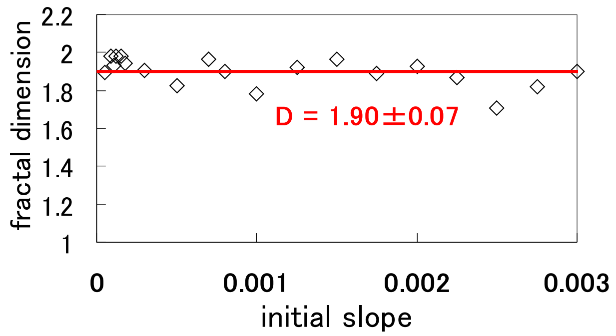

Fig. 4. Fractal dimension as a function of the initial slope.

As an initial condition, we add a minute disturbance

$$\tilde{z}(x, y) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} a_{ij} \sin \frac{2\pi i x}{L_x} \cos \frac{2\pi j y}{L_y} \qquad (10)$$

to the inclined plane $z(x, y) = z(0, 0) + S_0 y$, where S_0 is the initial slope and a_{ij} is a uniform random factor with a maximum value of 0.0001. Note that $\tilde{z}$ has some smoothness because of omitting high wave number modes. A sheet flow with the depth h_0 and zero velocity are set on the surface initially.

We use the periodic boundary conditions of u, v, h, and $z(x, y) - S_0 y$ in the x and y directions with periods $L_x = N_x \Delta x$ and $L_y = N_y \Delta y$. The values of the parameters are listed in Table 1.

4. Simulation Results

Figure 2 shows the simulation result for the initial value of the slope 0.0001. A complex pattern appears gradually as time passes; finally, a steady pattern is obtained. We convert the data of elevation z, shown in Fig. 2(4), into a binary image in order to calculate the fractal dimesion D using the box couting method. The fractal dimension D of this pattern becomes constant for large time, and the time-averaged value of D is about 1.87 for $t \geq 1100$ (Fig. 3). Moreover, the dimension remains almost constant at 1.90 when the initial slope is changed (Fig. 4). This corresponds to the phenomenon in which similar branching patterns appear in various geographical features. However, when the initial slope is smaller than 0.00005, the pattern is not clearly formed because erosion does not occur and the fractal dimension is not computable. In contrast, when the initial slope is steeper than 0.005 the pattern consists of straight streams; a complex pattern is not formed because of the large gravitational force and thus the fractal dimension is one.

Next, we consider the relationship between the longest stream length L and the drainage area A. The longest stream length is easily determined from the binary image. The drainage area for the longest stream is calculated as follows. First, a marker is placed on every grid point and the heights of eight neighbors are compared. Next, the markers are moved to the lowest point among the eight neighbors, and this procedure is repeated. In most cases, the marker reaches the outlet, while in a few cases, it stops before it reaches the outlet. In any case, one counts the marker in the drainage area if it is in the drainage basin of the stream considered. Finally, the drainage area A is obtained as the number of markers in the area related to the stream considered. As shown in Fig. 5, this result is consistent with Hack's law (1). The discrepancy between Hack's law and the simulation results for $\log L \geq 1.6$ arises due to the periodic boundary condition in the y direction. Since the size of region in the y drection L_y is 51.2, i.e. $\log L_y = 1.7$, this discrepancy is reasonable when L is large. Note that Fig. 5 is plotted for various initial slopes from 0.00005 to 0.003.

5. Conclusion

In this study, we propose a simple model for determining landscape evolution; this model employs shallow water equations and the mass conservation of sediment. In the simulation, this model generates steady pattern of river network. Numerical simulation shows that the fractal dimension of the river pattern is 1.90, which is independent of the initial slope and is close to the value of 1.7∼1.9 in the geographical features. This result is in good agreement with Hack's law, and hence, the proposed model is appropriate for pattern generation. Therefore, it is clarified that the mechansm responsible for the complex pattern is mass movement by fluid motion. The present model is simple, but rather impractical. However, more realistic effects such as non-periodic conditions and precipitation can be easily incorporated into the model. In future, we intend to explore the precise mechanism of fractal patterns and Hack's law.