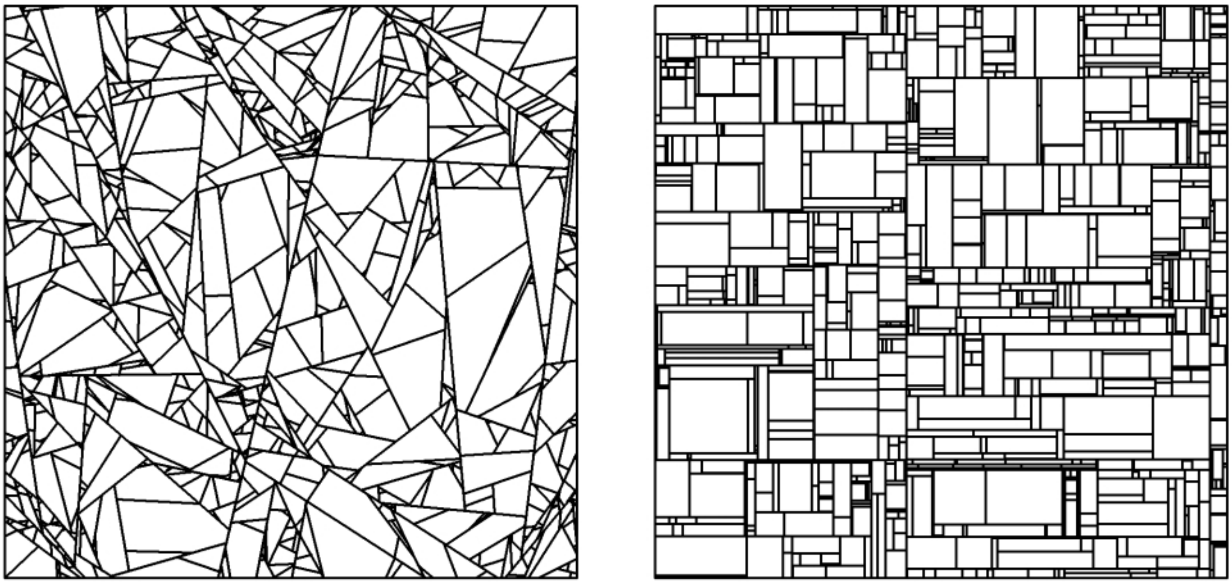

Fig. 2. Realizations of planar stationary STIT tessellations with different directional distributions (kindly provided by Joachim Ohser).

directional distribution or the distribution of the direction of the typical K, I- or J-segment to be the distribution of the line in $\mathcal{H}$ parallel to the line through the typical K-, I- or J-segment, respectively.

With these notions and notation we can now recall the main result from Mecke (2009):

The directional distribution of the typical I-segment has the density $\frac{1}{\zeta_\kappa} s_\kappa(\cdot)$ with respect to κ and the joint distribution of direction and length of the typical I-segment has the density

$$(h, x) \mapsto \frac{2}{\zeta_\kappa s_\kappa(h)} \int_0^{s_\kappa(h)} t^2 e^{-tx} dt, \quad h \in \mathcal{H}, \ x > 0$$

with respect to the product measure $\kappa \otimes \ell_+$, where ℓ_+ stands for the Lebesgue measure on the positive real half-axis.

For completeness and for later reference, let us recall the concept of *iteration of tessellations*. Let Φ be a homogeneous random planar tessellation with cells C_k, $k \in \mathbb{N}$, and let $(\Phi_k)_{k\in\mathbb{N}}$ a family of independent and identically distributed random tessellation, which is also independent of Φ and for which each Φ_k has the same distribution as Φ. We define the iteration of Φ with the the sequence $(\Phi_k)_{k\in\mathbb{N}}$ to be the random tessellation

$$\Phi \cup \bigcup_{k\in\mathbb{N}} (\Phi_k \cap C_k).$$

The resulting tessellation will be denoted by $\Phi \circledcirc \Phi$. The idea behind the definition is to associate to each cell C_k of the frame tessellation Φ a component tessellation Φ_k and to subdivide each cell C_k by its component tessellation Φ_k, i.e. to make local superpositions of the tessellations Φ and Φ_k inside the cells C_k. In this sense, a homogeneous random tessellation Φ is called iteration stable, if $2(\Phi \circledcirc \Phi)$ has the same distribution as Φ, where $2(\Phi \circledcirc \Phi)$ stands for the random tessellation $\Phi \circledcirc \Phi$ dilated by a factor 2. In other words, Φ is iteration stable if its distribution does not change under *rescaled* iteration. It was shown in Nagel and Weiss (2005) that for finite areas, the class of STIT tessellations as constructed in the Introduction is exactly the same as the class of iteration stable random tessellations. The stochastic stability of STIT tessellations under rescaled iteration will be the crucial tool to derive a balance equation for a certain distribution function related to the length and directional distribution of the typical K-segment below.

3. The Result for the Typical K-Segment

We establish in this section a similar formula for the joint density of direction and length of the typical K-segment as that one for the typical I-segment and we will use to this end the technique developed in Mecke (2009). We fix from now on a homogeneous random STIT tessellation in the plane with edge length intensity $L_A > 0$ and directional measure κ. From κ we can derive a translation invariant measure on the space $[\mathcal{G}, \mathfrak{G}]$ of all lines in the plane by putting $\mu := \kappa \otimes \ell_+$, where we have used the parametrization of a line $g \in \mathcal{G}$ by its direction $r(g) \in \mathcal{H}$—the factor κ—and its distance from the origin—the factor ℓ_+. We define now a new length measure l_κ in the plane by

$$l_\kappa(s) := \mu\{g \in \mathcal{G} : g \cap s \neq \emptyset\} = s_\kappa(h) l(s)$$

for any line segment $s \subset \mathbb{R}^2$, where $h \in \mathcal{H}$ is parallel to $g \in \mathcal{G}$, $s \subset g$, and l denotes the usual Euclidean length. We will call $l_\kappa(s)$ the κ**-length** of the segment s. Observe, that the intersection property of STIT tessellations mentioned above can now be formulated as follows: *The intersection of a homogeneous random STIT tessellation having directional measure κ with a line $g \in \mathcal{G}$ is a homogeneous random point process on g with κ-intensity* 1, *i.e. the mean number of points per unit κ-length equals* 1.

Let Φ be the previously fixed random STIT tessellation