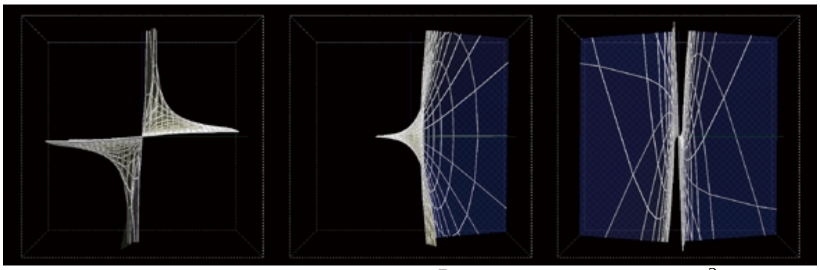
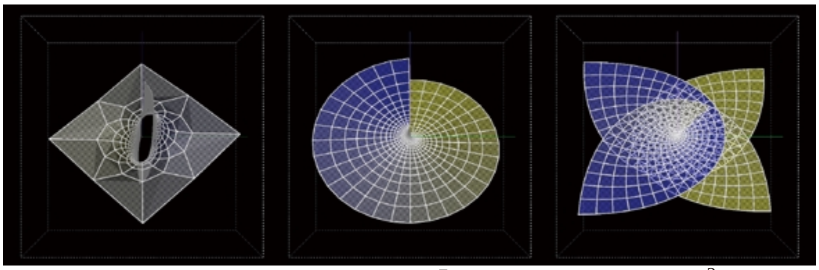


$f(Z)=1/Z$ (a) $p_f(0, 0, 0, 4)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=e^Z$ (b) $p_f(0, 0, 0, 10)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=Z^3$ (c) $p_f(0, 0, 0, 30)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

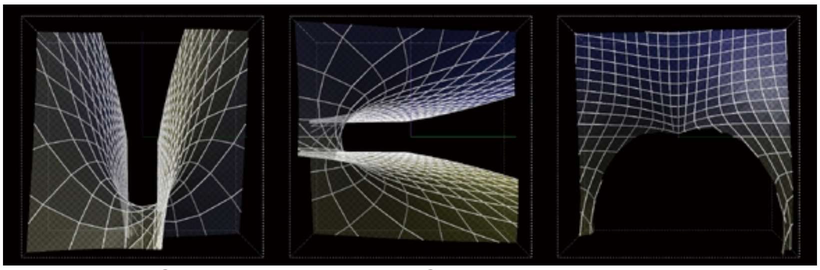


$f(Z)=1/Z$ (d) $p_f(4, 4, 0, 0)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=e^Z$ (e) $p_f(20, 20, 0, 0)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=Z^3$ (f) $p_f(50, 50, 0, 0)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

Fig. 4. Mapping structures of complex functions by perspective projection.



$f(Z)=Z^2$-1 (a) $p_f(0, 0, 0, 1.5)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=Z^2$+1 (b) $p_f(0, 0, 0, 1.5)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

$f(Z)=1/Z$ (c) $p_f(0, 0, 0, 1.5)$, $p_a(0, 0, 0, 0)$, k 0.5, h 0.5, f 200

Fig. 5. Local structures of complex functions by perspective projection.

(b), and (c), the input complex plane and the real part of the output complex plane are displayed on the 3-D screen. Figures 6(d), (e), and (f) show the input complex plane and the imaginary part of the output complex plane. This representation in 3-D space is useful for building up organized knowledge from 4-D data.

A slice operation is realized by locating the 3-D screen near the background hyperplane. Figure 7 shows mapping structures and roots of complex functions clipped by a slice operation. In this representation, the section becomes a 1-D line. When we observe the complex function from the 4-D eye-point on the w_w-axis in the 4-D space by a slice operation, we can see the original real function, including the imaginary part, on the 3-D screen (see Figs. 7(a), (b), and (c)). Although the real and imaginary parts of an arbitrary equation are not generally represented in 2-D space, we can have visual contact with the real and imaginary parts of an arbitrary equation in the system at the same time. As indicated by the white circles in Figs. 7(d), (e), and (f), by focusing on the input complex plane, we can have perfect visual contact with not only the real and imaginary roots, but also the conjugate roots, without using numerical mathematics such as quadratic formulas and Newton methods.

4.3 Visualizations of functions with three variables

If there are scalar values in a 3-D space, the 3-D scalar function $f(G)$ is defined as the relationship between the three inputs $G = (x, y, z)$ and the one output $f(G) = w(x, y, z)$. For example, the 3-D Gaussian function $f(G) = w(x, y, z) = \exp\{-(x^2 + y^2 + z^2)\}$ is a 3-D scalar potential, and corresponds to the 4-D information