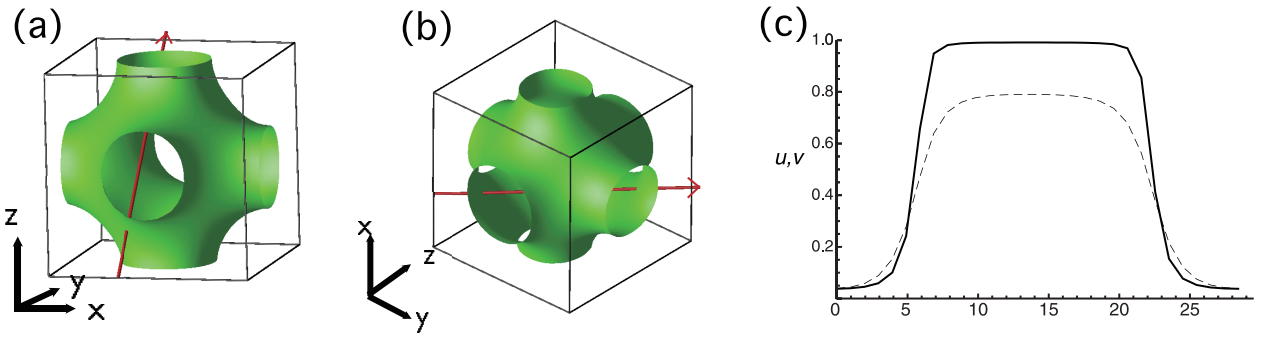

Fig. 3. (a), (b) Isosurface of P surface obtained by Eqs. (1) and (2) for $u_0 = 0.45$ from different view directions. (c) Spatial variations of concentrations u (solid line) and c (dotted line) along the red arrow in (a) and (b).

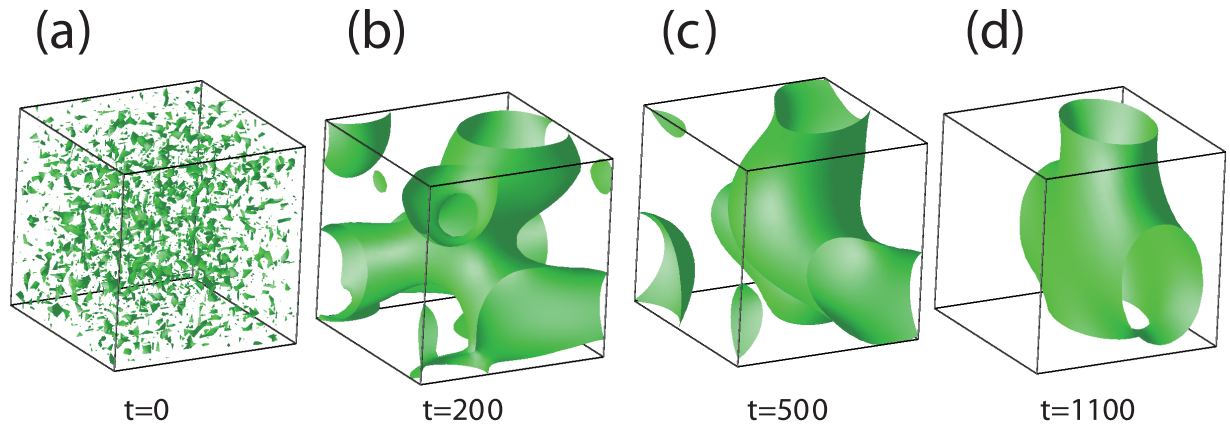

Fig. 4. Time evolution of perforated lamellae for $u_0 = 0.35$, and $L = \delta x N$ with $\delta x = 0.63$ and $N = 0.32$. The other parameters are same as in Fig. 1. In order to make the initial randomness visible, the domains in (a) represent the isosurface of $u = 0.35075$, whereas those in (b)–(d) represent the isosurface of $u = 0.35$.

cause Eqs. (1) and (2) are nonvariational with no Lyapunov functional. Here, we employ two methods to examine the stability as follows.

One of the methods is to derive approximately a Lyapunov functional for Eqs. (1) and (2) (Chavanis, 2003). In the limit $D_c \to +\infty$, one may set $\partial c/\partial t = 0$ in Eq. (2), so that Eq. (1) can be viewed as a nonlinear mean-field Fokker-Planck equation associated with a Langevin dynamics of the form

$$\frac{d\vec{r}}{dt} = (1-u)\nabla c + \sqrt{2D_u}\vec{R}(t), \tag{3}$$

where $\vec{R}(t)$ is white noise. Equation (3) describes a point organism $\vec{r}$ performing a random walk biased in the direction of a drift velocity proportional to the local density of u and to the local gradient of c. In this situation, the physical temperature $T = \beta D_u$ defined by the Einstein relation is fixed instead of the energy since $D_u \propto T$ (Chavanis, 2003).

Here, we can consider the Helmholz free energy as a Lyapunov functional,

$$F = -\frac{1}{2}\int u c d\vec{r} + T\int \{u\ln u + (1-u)\ln(1-u)\}\, d\vec{r}, \tag{4}$$

where the first term represents the self-interaction and the second term expresses the entropy. Chavanis (2003) used the entropy of Felmi-Dirac type, because the density always remains bounded by parabolic terms in Eq. (1) as shown in Fig. 3(c). In Chavanis (2003), it was shown $\dot{F} \leq 0$, which is similar to the proper version of the H-theorem of the canonical ensemble.

We evaluate the Helmholz free energy of SP, CY, PL, P, LM and uniform distributions by substituting the asymptotic values of u and c directly into Eq. (4). These asymptotic values were prepared as follows.

For each parameter u_0 and λ, the distribution of u is estimated as $u(\vec{r}) = 1/(1+e^{B\vec{r}+A})$ corresponding to $\dot{F} = 0$ (Chavanis, 2003). For example, in the case of the SP, numerical calculations are started from the distributions such as

$$u(\vec{r}) = \frac{1}{1+\exp\left[B\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}\right]}, \tag{5}$$

where (x_0, y_0, z_0) is its center. B is the slope of the interface between cell and non-cell areas. The distribution of $c(\vec{r})$ can be expressed numerically by the solution of Helmholz equation using Bessel functions. Carrying out the numerical simulation, in which these distributions are employed as initial conditions, for a sufficient long time such that the distributions would no longer change, we can obtain the equilibrium distributions. Introducing the each pattern (SP,