

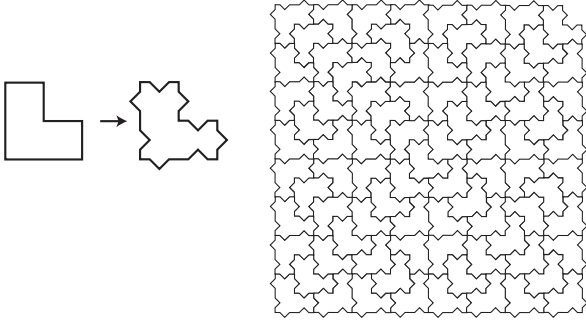


Fig. A.1. Tiling of one prototile.

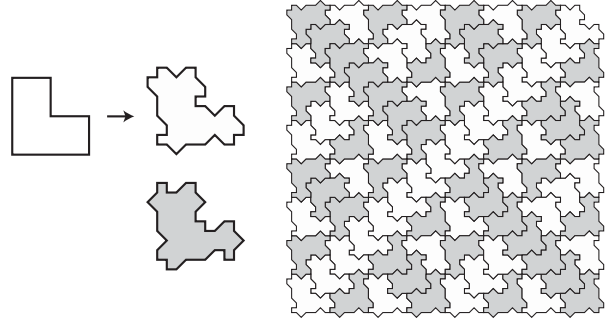


Fig. A.4. Tiling of two prototiles (5,10).

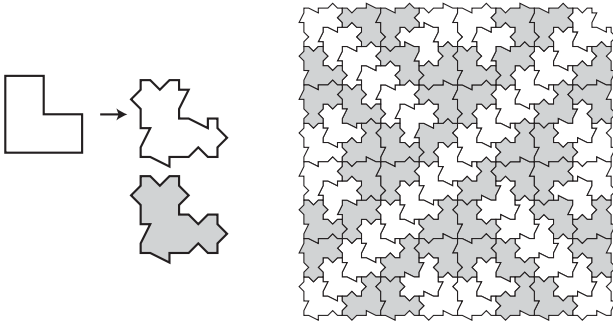


Fig. A.2. Tiling of two prototiles, no.5.

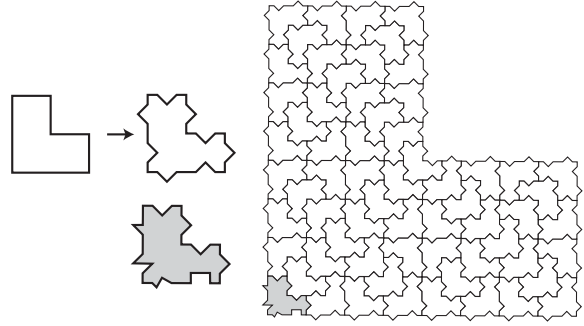


Fig. A.5. Tiling of two prototiles (0,2).

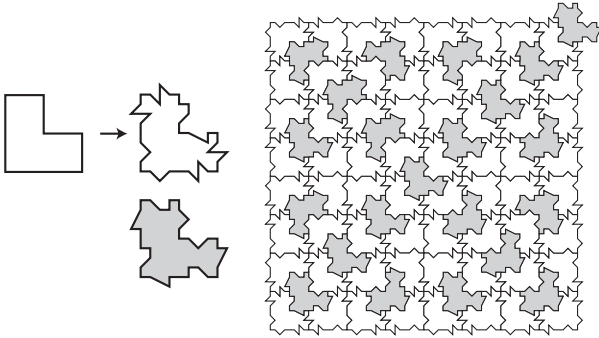


Fig. A.3. Tiling of two prototiles, no.8.

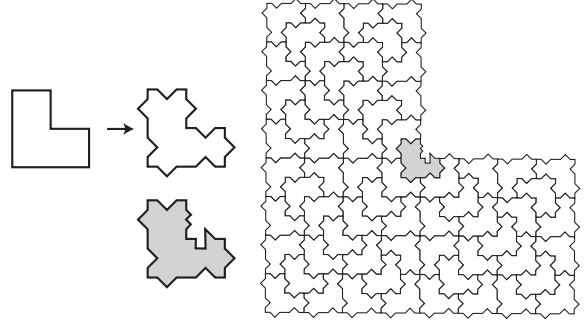


Fig. A.6. Tiling of two prototiles (0,8).

$$\frac{e^{(s-1)} = i^{(s-1)} = k^{(s-1)} = o^{(s-1)}}{b^{(s-1)} = d^{(s-1)} = h^{(s-1)} = n^{(s-1)}}, \text{ then}$$

$$\frac{a^{(s)} = c^{(s)} = g^{(s)} = m^{(s)}}{f^{(s)} = j^{(s)} = l^{(s)} = p^{(s)}}, \frac{e^{(s)} = i^{(s)} = k^{(s)} = o^{(s)}}{b^{(s)} = d^{(s)} = h^{(s)} = n^{(s)}}.$$

Proof: For example, $p^{(s-1)} = l^{(s-1)}$ and $a^{(s-1)} = m^{(s-1)}$ implies $a^{(s)} = p^{(s-1)}a^{(s-1)} = l^{(s-1)}m^{(s-1)} = c^{(s)}$. Other relations are shown in a similar way. From this lemma, $(s+1)$ -spreads $\alpha^{(s+1)}$, $\beta^{(s+1)}$ exist for any s .

(3), (4), and (5) are shown in a similar way as (2).

REMARK 5.5 In Appendix, we show figures of these tilings. In $(0, 2)$, $(0, 8)$ tilings, $\beta^{(s)}$ contains only one tile β . This means that these tilings are equivalent to a tiling of α as tilings.

Acknowledgments. The authors would like to thank Prof Koukichi Sugihara for his giving them such an interesting topics. The authors also thank the referee for his kind and useful advice.

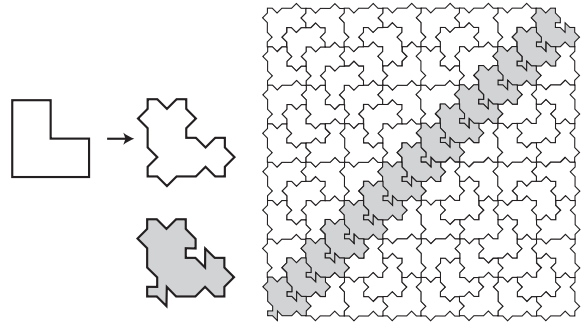


Fig. A.7. Tiling of two prototiles (0,10).

Appendix A.

From Figs. A.1 to A.7 are pictures of tilings appearing in Theorems A, B, and C.

Appendix B.

For a protoset $\mathcal{S}$, if **any** tiling by $\mathcal{S}$ has no periodicity