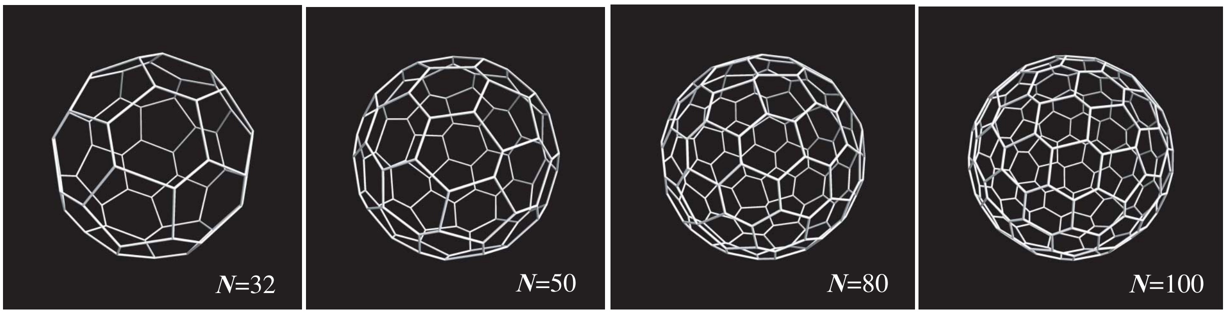

Fig. 9. Examples of polyhedrons obtained from the MPAVP ($n = 1$) model ($N = 32$, 50, 80, and 100).

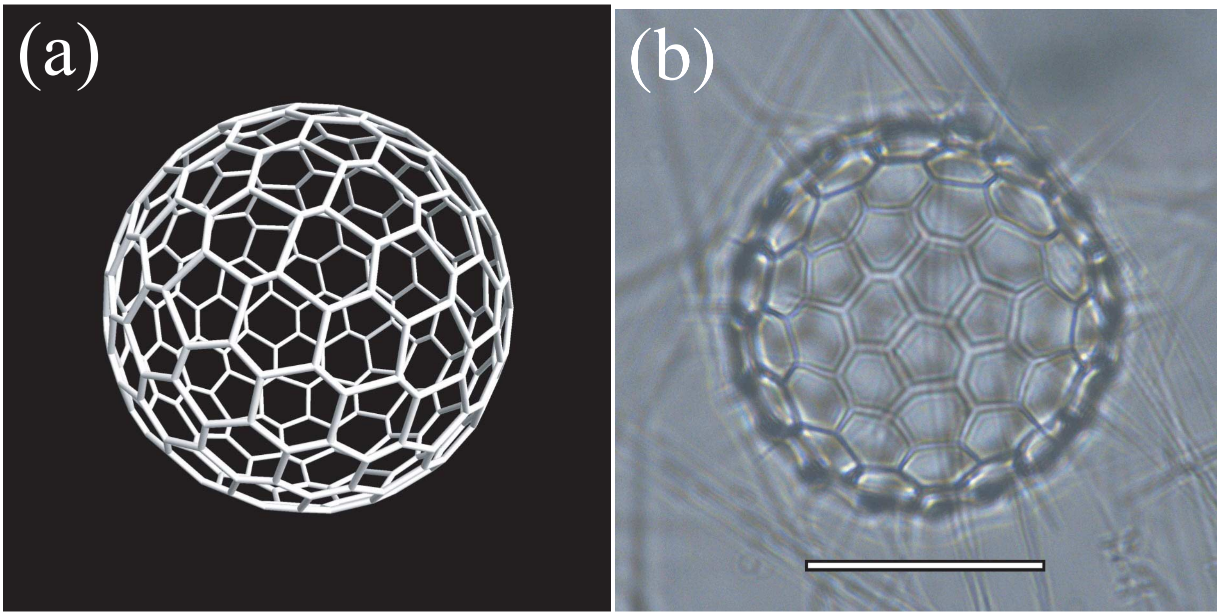

Fig. 10. Comparison of an example of a MPVAP ($N = 138$ and $n = 1$) with almost-regular type of spherical radiolarian (*Acanthosphaera circopora* Popofsky). Scale bar indicates 50 μm.

3. Results

Figure 5 shows some examples of convex hull of generators randomly allocated on the sphere for different numbers of points N. The obtained polyhedrons have two features: most vertices have mainly 6 associated edges, and almost all the faces are triangles. These results are consistent with Euler's theorem of convex polyhedrons. From the viewpoint of cost minimization of skeleton-forming materials, the resultant skeletons are clearly not optimal. Furthermore, such polyhedrons are not similar to any types of real spherical radiolaria. From this, we conclude that this model is not appropriate for spherical radiolaria.

Figure 6 shows examples of numerical results for the approximated Voronoi polyhedrons generated by randomly allocated generators on sphere (RAVP). The generator distributions of the examples were same ones with the point distributions of examples in Fig. 5. Because of the duality, the polyhedrons consist mainly of hexagons with vertices of degree three. one.

Comparison of a numerical result for $N = 1540$ and a real sample (*Cyrtidosphaera reticulata* Haeckel) is shown in Fig. 7. The value 1540 was estimated from the 2D image in Fig. 1(a) as mentioned above. The result was similar to a type of radiolarian that has a mesh-like skeleton intuitively.

In order to consider the features of the random approximated Voronoi polygons (RAVPs), we compared the mean total edge lengths of the polyhedrons with those of convex hulls for each N. We generated 100 samples of spherical random points and calculated their convex hulls and RAVPs. Then, we obtained mean total edge lengths for both the convex hulls and RAVPs. The results are shown in Fig. 8. The mean total edge length of the RAVP is smaller than that of the convex hull for all N's except for $N = 4$ and 5. For small values of N, oblate polyhedrons are produced frequently so that the mean value of the total edge length of RAVP can also be smaller than those of random convex hulls.

Examples of the resulting polyhedrons of MPAVP, our another model, are shown in Fig. 9. The obtained polyhedrons consist only of nearly regular pentagons and hexagons. We also observed a striking similarity between the $N = 138$ polyhedrons and a real spherical radiolaria