Table 5. Uncertain cases of whether a convex pentagon can generate an edge-to-edge tiling.

Type of Set	Sub-case of v_1	Sub-case of v_2	Conditions of convex pentagon† Angle conditions	Edge conditions	Cyclic-edge-type	The simplest node condition
$\mathbf{G}_1$	ABD-1	ABD-1	$A+B+D=360^\circ$	$a=e, c=d$	[11223]	
	ABD-1	ABD-3	$A+B+D=360^\circ$	$a=e, b=c=d$	[11122]	
$\mathbf{G}_2$	AAB-1	BBC-1	$2A+B=2B+C=360^\circ$	$a=b=c=d$	[11112]	N
	AAB-1	BBE-1	$2A+B=2B+E=360^\circ$	$a=b=c=e$	[11112]	N
	AAB-1	CCD-2	$2A+B=2C+D=360^\circ$	$a=b=c, d=e$	[11122]	
	AAB-1	DDA-1	$2A+B=2D+A=360^\circ$	$a=b=c=d$	[11112]	N
	AAB-1	DDA-2	$2A+B=2D+A=360^\circ$	$a=b=c=e$	[11112]	N
	AAB-1	DDE-2	$2A+B=2D+E=360^\circ$	$a=b=c=e$	[11112]	
	AAB-1	EEA-1	$2A+B=2E+A=360^\circ$	$a=b=c=e$	[11112]	N
	AAB-1	EEA-2	$2A+B=2E+A=360^\circ$	$a=b=c$	[11123]	N
	AAB-2	AAD-1	$2A+B=360^\circ, B=D$	$a=d=e, b=c$	[11122]	N
	AAB-2	BBD-1	$2A+B=2B+D=360^\circ$	$b=c=d=e$	[11112]	N
	AAB-2	CCD-2	$2A+B=2C+D=360^\circ$	$b=c, d=e$	[11223]	
	AAB-2	DDB-1	$A+B+D=360^\circ, A=D$	$b=c=d$	[11123]	
	AAB-2	DDC-1	$2A+B=2D+C=360^\circ$	$b=c=d$	[11123]	
	AAC-1	BBA-1	$2A+C=2B+A=360^\circ$	$a=b=c=d$	[11112]	N
	AAC-1	BBD-2	$2A+C=2B+D=360^\circ$	$a=c=d=e$	[11112]	
	AAC-1	DDB-1	$2A+C=2D+B=360^\circ$	$a=b=c=d$	[11112]	
	AAC-2	DDB-1	$2A+C=2D+B=360^\circ$	$b=c=d$	[11123]	
$\mathbf{G}_3$	AAB-1	EAC-1	$2A+B=E+A+C=360^\circ$	$a=b=c, d=e$	[11122]	N
	AAB-2	ABD-1	$A+B+D=360^\circ, A=D$	$a=e, b=c=d$	[11122]	N
	AAB-2	CDA-1	$2A+B=C+D+A=360^\circ$	$a=e, b=c$	[11223]	N
	AAB-2	CDA-3	$2A+B=C+D+A=360^\circ$	$a=d=e, b=c$	[11122]	N
	AAB-2	CDA-5	$2A+B=C+D+A=360^\circ$	$a=e, b=c=d$	[11122]	N
	AAB-2	DEB-7	$2A+B=D+E+B=360^\circ$	$a=d, b=c=e$	[11212]	
	AAB-2	EAC-1	$2A+B=E+A+C=360^\circ$	$b=c, d=e$	[11223]	N
$\mathbf{G}_4$	ABD-1	AAA	$A=120^\circ, A+B+D=360^\circ$	$a=b=e, c=d$	[11122]	N
$\mathbf{G}_5$	AAB-1	DDD	$D=120^\circ, 2A+B=360^\circ$	$a=b=c, d=e$	[11122]	N
	AAB-1	EEE	$E=120^\circ, 2A+B=360^\circ$	$a=b=c=e$	[11112]	N
	AAB-2	DDD	$D=120^\circ, 2A+B=360^\circ$	$b=c, d=e$	[11223]	N
	AAB-2	EEE	$E=120^\circ, 2A+B=360^\circ$	$a=e, b=c$	[11223]	N
	AAC-1	BBB	$B=120^\circ, 2A+C=360^\circ$	$a=b=c=d$	[11112]	N
	AAC-2	BBB	$B=120^\circ, 2A+C=360^\circ$	$b=c=d$	[11123]	N
	AAC-2	EEE	$E=120^\circ, 2A+C=360^\circ$	$a=e, b=c=d$	[11122]	N

†: The notation of the conditions follows the present classification rule.

Example 5.2. Case that v_1 is *AAB*-1 and v_2 is *EED*-1. This is the case that $2A+B=2E+D=360^\circ$, $a=b=c$, $d=e$. Suppose that such convex pentagon exists. Let M be the midpoint of the diagonal AC. Then, since $a=b$, we have $BM \perp AC$, and since $2A+B=360^\circ$, $BM \parallel AE$. Hence $AE \perp AC$. Similarly, we have $AE \perp CE$. Therefore, $\angle ACE = 0^\circ$, a contradiction.

By examining each of 465 patterns analogously to the above, the 465 cases are classified into four categories: (i) a convex pentagon cannot exist; (ii) a convex pentagon cannot generate an edge-to-edge tiling (even if it exists); (iii) a convex pentagon belongs to at least one of type 1, type 2, type 4, type 5, type 6, type 7, type 8, or type 9 (if it exists); and (iv) uncertain case (unknown whether a convex pentagon can generate an edge-to-edge tiling). At present, there are 34 uncertain cases remained. These 34 uncertain cases are listed in Table 5. I am working on these 34 patterns now, and they will be settled in near future.

6. Proofs of Theorems 2, 3, 4

6.1 Proof of Theorem 2

Pentagons can be classified by the number of equal edges and their positions. In the following, the edges are designated symbolically in $1, 2, \ldots$ in cyclic (anticlockwise) order, with the same symbol for equal edges. Mirror-reflections are excluded. Beginning with equilateral pentagons, followed by those with four equal edges, etc., they are classified into 12 cyclic-edge-types: [11111], [11112], [11122], [11212], [11123], [11213], [11223], [11232], [12123], [11234], [12134], [12345] (Sugimoto and Ogawa, 2006).

The convex pentagons of cyclic-edge-types in the 465