

Fig. 1. Archimedean tilings.

close proximity to facilities. If customers are uniformly distributed and serviced by their nearest facility, the optimal facility location is  $(3^6)$ , as shown by Leamer (1968), Iri *et al.* (1984), and Du *et al.* (1999). If some of the existing facilities are closed and customers are serviced by their second nearest facility, however, other patterns of facilities can be optimal. The distances in Archimedean tilings will thus give an insight into facility location problems with closing of facilities. As an application to location analysis, we consider bi-objective problems where two distances are minimized. We then present Pareto optimal solutions for the problems. Pareto optimal solutions are such that no other solution is superior to them and have been used in multi-criteria facility location problems (Nickel *et al.*, 2005; Farahani *et al.*, 2010).

The remainder of this paper is organized as follows. The next section derives the average distances to the first and second nearest vertices of Archimedean tilings. The following section examines the maximum distances. The penultimate section provides an application to location analysis. The final section presents concluding remarks.



Fig. 2. Average distance in a right triangle.

## 2. Average Distance

Let  $E(R_1)$  and  $E(R_2)$  be the average distances from a random point on a plane to the first and second nearest vertices, respectively. In this section, we derive  $E(R_1)$  and  $E(R_2)$  in Archimedean tilings.

The average distance from a random point in a right triangle to a vertex was derived by Koshizuka and Ohsawa (1983). Let *R* be the distance from a random point in the right triangle with side lengths *a* and *b* (a > b) to the vertex *O*, as shown in Fig. 2. The sum of the distances T(a, b) is